

Predictive Attitude Estimation Using Global Positioning System Signals

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Abstract

In this paper, a new algorithm is developed for attitude estimation using Global Positioning System (GPS) signals. The new algorithm is based on a predictive filtering scheme designed for spacecraft without rate measuring devices. The major advantage of this new algorithm over traditional Kalman filter approaches is that the model error is not assumed to be represented by an unbiased Gaussian noise process with known covariance, but instead is determined during the estimation process. This is achieved by simultaneously solving system optimality conditions and an output error constraint. This approach is well suited for GPS attitude estimation since some error sources that contribute to attitude inaccuracy, such as signal multipath, are known to be non-Gaussian processes. Also, the predictive filter scheme can use either GPS signals or vector observations or a combination of both for attitude estimation, so that performance characteristics can be maintained during periods of GPS attitude sensor outage. The performance of the new algorithm is tested using flight data from the REX-II spacecraft. Results are shown using the predictive filter to estimate the attitude from both GPS signals and magnetometer measurements, and comparing that solution to a magnetometer-only based solution. Results using the new estimation algorithm indicate that GPS-based solutions are verified to within 2 degrees using the magnetometer cross-check for the REX-II spacecraft. GPS attitude accuracy of better than 1 degree is expected per axis, but cannot be reliably proven due to inaccuracies in the magnetic field model.

Introduction

The concept of using phase difference measurements from GPS receivers for three-axis attitude determination has been successfully proven on many systems in the past [1-3]. However, to this date only a handful of these experiments have been tested on spacecraft. One of the first space-based applications was flown on the RADCAL (RADAR CALibration) spacecraft [4], which demonstrated GPS attitude determination using post-processed measurements. To obtain maximum GPS visibility, and to reduce signal interference due to multipath reflection, GPS patch antennas were placed on the top surface of the spacecraft bus. Although the antenna baselines were short for attitude determination, attitude accuracy of about 2 degrees per axis 3σ was achieved for a 0.67 meter antenna separation. Another experiment, Crista-SPAS [5], provided the first on-orbit demonstration of real-time attitude determination. The spacecraft contained an accurate gyro reference, but the coordinate frame alignment was not measured relative to the GPS attitude reference frame, which means that discrepancies between the two reference frames might account for slightly different measurements from the two systems. Over the course of the experiment the two sets of attitude solutions agreed to within 2 degrees, which is thought to be within the alignment

tolerance of the two reference frames. The first extended real-time GPS based attitude determination mission was flown on the REX-II spacecraft [6], which furthermore tested actual attitude control using GPS measurements.

The differential carrier phase error has a standard deviation of about 5 mm, which is a small fraction of the 19 cm standard wavelength [7]. However, many error sources can significantly contribute to attitude inaccuracy. These include: integer ambiguity resolution of the GPS carrier, reflections of the GPS carrier from the environment surrounding the receiver (multipath), line bias errors between receivers, receiver motion due to external distortions (e.g., thermal disturbance effects), constellation availability, tropospheric refraction, and cross-talk errors. The most significant error source and most difficult to overcome is multipath [3]. In fact, multipath effects can be a major driving source for the location of the GPS antennas on a vehicle. Many techniques have been presented to resolve the integer ambiguity problem [8]. An approach using an H_∞ -type filter has been shown to improve attitude determination performance with line biases [9]. Other error sources, such as tropospheric refraction, can be modeled out for relatively short baselines (less than three meters) [3].

Three-axis attitude solutions may be found using both deterministic (point-by-point) and estimator-based (i.e., propagation of a dynamic model) techniques. The main advantage of using estimator-based techniques, such as the Kalman filter, is that the attitude can be found using a single baseline or sightline, as long as there is sufficient vehicle motion to couple errors along the unobservable baseline direction into two orthogonal axes [10]. Another advantage is that some error sources, such as line biases, can be estimated concurrently with the attitude. Fujikawa and Zimbelman [10] developed a Kalman filter using GPS signals to successfully estimate the attitude of a spacecraft and line bias using one baseline. The main advantage of deterministic methods is that an initial estimate of the attitude is not required to develop a solution. Also, deterministic methods are usually computationally more efficient as compared to estimator-based techniques. Deterministic methods can also provide an initial estimate for a filter. Choosing between deterministic and estimator-based techniques usually depends on the particular application and requirements.

In this paper a new algorithm is developed which is used to estimate the attitude of a spacecraft using GPS phase difference measurements and a dynamic model. The new algorithm is based on a predictive filtering scheme first introduced by Crassidis and Markley [11]. One of the difficulties demonstrated by Fujikawa and Zimbelman [10], using a six state Kalman filter for GPS attitude estimation, is that large attitude deviations are possible due to the effect of external torque disturbances. To overcome this difficulty the state model was appended to incorporate torque estimation. The algorithm developed in this paper, unlike the Kalman filter, does not assume that the external torque is modeled by a zero-mean Gaussian process. Instead, it is automatically determined during the estimation process, without using an appended state model. Therefore, the new algorithm provides a more practical method for attitude estimation.

The organization of this paper proceeds as follows. First, a summary of the spacecraft attitude kinematics, dynamics, and sensor models is shown. Then, a brief review of the predictive filter for nonlinear systems is shown. Next, a predictive filter is developed for the purpose of attitude estimation using GPS measurement signals and a dynamic model. This approach estimates the optimal spacecraft attitude in real-time by minimizing a quadratic cost function consisting of a measurement residual term and a model error term. Two algorithms are shown. The first uses the GPS phase difference measurements for attitude estimation, and the second uses GPS-found attitudes from deterministic approaches for attitude estimation. Finally, the predictive filter is used to estimate and verify the attitude of REX-II in order to demonstrate the usefulness of this algorithm. Case comparisons are made with respect to a magnetometer-only based solution using another predictive filter approach.

Attitude Kinematics and Dynamics

In this section, a brief review of the kinematic and dynamic equations of motion for a three-axis stabilized spacecraft is shown. The attitude is assumed to be represented by the quaternion, defined as

$$\underline{q} \equiv \begin{bmatrix} q_{13} \\ q_4 \end{bmatrix} \quad (1)$$

with

$$\underline{q}_{13} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \hat{n} \sin(\theta/2) \quad (2a)$$

$$q_4 = \cos(\theta/2) \quad (2b)$$

where \hat{n} is a unit vector corresponding to the axis of rotation and θ is the angle of rotation. The quaternion kinematic equations of motion are derived by using the spacecraft's angular velocity ($\underline{\omega}$), given by

$$\dot{\underline{q}} = \frac{1}{2} \Omega(\underline{\omega}) \underline{q} = \frac{1}{2} \Xi(\underline{q}) \underline{\omega} \quad (3)$$

where $\Omega(\underline{\omega})$ and $\Xi(\underline{q})$ are defined as

$$\Omega(\underline{\omega}) \equiv \begin{bmatrix} -[\underline{\omega} \times] & \vdots & \underline{\omega} \\ \dots & \vdots & \dots \\ -\underline{\omega}^T & \vdots & 0 \end{bmatrix} \quad (4a)$$

$$\Xi(\underline{q}) \equiv \begin{bmatrix} q_4 I_{3 \times 3} + [\underline{q}_{13} \times] \\ \dots \\ -\underline{q}_{13}^T \end{bmatrix} \quad (4b)$$

where $I_{3 \times 3}$ is a 3×3 identity matrix. The 3×3 dimensional matrices $[\underline{\omega} \times]$ and $[\underline{q}_{13} \times]$ are referred to as cross product matrices since $\underline{a} \times \underline{b} = [\underline{a} \times] \underline{b}$, with

$$[\underline{a} \times] \equiv \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad (5)$$

The quaternion obeys a single constraint, given by

$$\underline{q}^T \underline{q} = \underline{q}_{13}^T \underline{q}_{13} + q_4^2 = 1 \quad (6)$$

The dynamic equations of motion, also known as Euler's equations, for a rotating spacecraft are given by [12]

$$\dot{\underline{H}} = \underline{N} - \underline{\omega} \times \underline{H} \quad (7)$$

where \underline{H} is the total angular momentum, \underline{N} is the total external torque (which includes, e.g., control torques, aerodynamic drag torques, solar pressure torques, etc.), and J is the inertia matrix of the spacecraft. If reaction or momentum wheels are used on the spacecraft, the total angular momentum is given by

$$\underline{H} = J \underline{\omega} + \underline{h} \quad (8)$$

where \underline{h} is the total angular momentum due to the wheels. Thus, Equation (7) can be re-written as

$$\dot{\underline{H}} = \underline{N} - \left[J^{-1}(\underline{H} - \underline{h}) \right] \times \underline{H} \quad (9)$$

Also, from Equations (7) and (8) the following angular velocity form of Euler's equation can be used

$$J \dot{\underline{\omega}} = \underline{N} - \dot{\underline{h}} - \underline{\omega} \times (J \underline{\omega} + \underline{h}) \quad (10)$$

which involves the derivative of the wheel angular momentum.

GPS Sensor Model

In this section, a brief background of the GPS phase difference measurement is shown. The main measurement used for attitude determination is the phase difference of the GPS signal received from two antennas separated by a baseline. The principle of the wavefront angle and wavelength, which are used to develop a phase difference, is illustrated in Figure 1.

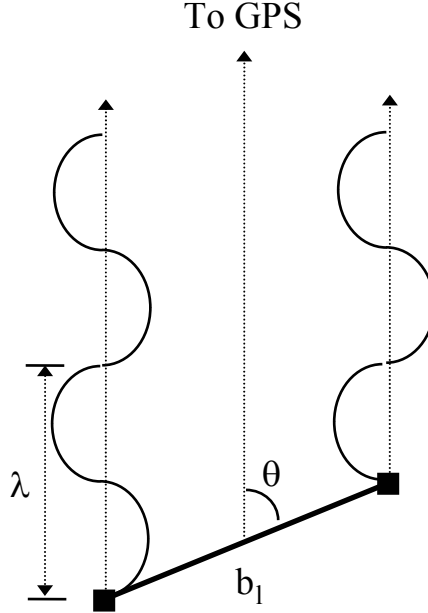


Figure 1 GPS Wavelength and Wavefront Angle

The phase difference measurement is obtained by

$$b_l \cos \theta = \lambda \left(n + \Delta \phi^0 / 2\pi \right) \quad (11)$$

where b_l is the baseline length, θ is the angle between the baseline and the line of sight to the GPS spacecraft, n is the number of integer wavelengths between two receivers, $\Delta\phi^0$ is the actual phase difference measurement, and λ is the wavelength of the GPS signal. The two GPS frequency carriers are L1 at 1575.42 MHz and L2 at 1227.6 MHz. As of this writing, non-military applications generally use the L1 frequency. Then, assuming no integer offset, we define a normalized phase difference measurement $\Delta\phi$ by

$$\Delta\phi \equiv \frac{\lambda \Delta\phi^0}{2\pi b_l} = \underline{b}^T A \underline{s} \quad (12)$$

where $\underline{s} \in R^3$ is the normalized line of sight vector to the GPS spacecraft in an inertial frame, $\underline{b} \in R^3$ is the normalized baseline vector, which is the relative position vector from one receiver to another, and the attitude matrix $A \in SO(3)$, the Lie group of 3×3 orthogonal matrices with determinant 1 (i.e., $A^T A = I_{3 \times 3}$ and $\det A = 1$). The attitude matrix is related to the quaternion by

$$A(\underline{q}) = -\Xi^T(\underline{q})\Psi(\underline{q}) \quad (13)$$

where

$$\Psi(\underline{q}) \equiv \begin{bmatrix} -q_4 I_{3 \times 3} + [\underline{q}_{13} \times] \\ \dots\dots\dots \\ \underline{q}_{13}^T \end{bmatrix} \quad (14)$$

and $\Xi(\underline{q})$ is given by Equation (4b).

GPS Attitude Estimation

In this section, a predictive filter for attitude estimation is developed using GPS measurements. First, a brief review of the nonlinear predictive filter is shown (see Ref. [11] for more details).

Predictive Filtering

In the nonlinear predictive filter it is assumed that the state and output estimates are given by a preliminary model and a to-be-determined model error vector, given by

$$\dot{\underline{\hat{x}}}(t) = \underline{f}(\underline{\hat{x}}(t), t) + G(t)\underline{d}(t) \quad (15a)$$

$$\underline{\hat{y}}(t) = \underline{c}(\underline{\hat{x}}(t), t) \quad (15b)$$

where \underline{f} is a $p \times 1$ model vector, $\underline{\hat{x}}(t)$ is a $p \times 1$ state estimate vector, $\underline{d}(t)$ is a $l \times 1$ model error vector, $G(t)$ is a $p \times l$ model-error distribution matrix, \underline{c} is a $m \times 1$ measurement model vector, and $\underline{\hat{y}}(t)$ is a $m \times 1$ estimated output vector. State-observable discrete measurements are assumed for Equation (15b) in the following form

$$\underline{\tilde{y}}(t_k) = \underline{c}(\underline{x}(t_k), t_k) + \underline{v}(t_k) \quad (16)$$

where $\underline{\tilde{y}}(t_k)$ is a $m \times 1$ measurement vector at time t_k , $\underline{x}(t_k)$ is the true state vector, and $\underline{v}(t_k)$ is a $m \times 1$ measurement noise vector which is assumed to be a zero-mean, Gaussian white-noise distributed process with

$$E\{\underline{v}(t_k)\} = \underline{0} \quad (17a)$$

$$E\{\underline{v}(t_k)\underline{v}^T(t_{k'})\} = R \delta_{kk'} \quad (17b)$$

where R is a $m \times m$ positive-definite covariance matrix.

A loss functional consisting of the weighted sum square of the measurement-minus-estimate residuals plus the weighted sum square of the model correction term is minimized, given by

$$J = \frac{1}{2} \{ \underline{\tilde{y}}(t_{k+1}) - \hat{\underline{y}}(t_{k+1}) \}^T R^{-1} \{ \underline{\tilde{y}}(t_{k+1}) - \hat{\underline{y}}(t_{k+1}) \} + \frac{1}{2} \underline{d}^T(t_k) W \underline{d}(t_k) \quad (18)$$

where W is a $l \times l$ weighting matrix. The necessary conditions for the minimization of Equation (18) lead to the following model error solution

$$\underline{d}(t_k) = - \left\{ \left[\Lambda(\Delta t) S(\hat{\underline{x}}_k) \right]^T R^{-1} \Lambda(\Delta t) S(\hat{\underline{x}}_k) + W \right\}^{-1} \left[\Lambda(\Delta t) S(\hat{\underline{x}}_k) \right]^T R^{-1} \left[\underline{z}(\hat{\underline{x}}_k, \Delta t) - \underline{\tilde{y}}(t_{k+1}) - \hat{\underline{y}}(t_k) \right] \quad (19)$$

where $\hat{\underline{x}}_k \equiv \hat{\underline{x}}(t_k)$, Δt is the measurement sampling interval, $S(\hat{\underline{x}})$ is a $m \times l$ dimensional matrix, and $\Lambda(\Delta t)$ is a $m \times m$ diagonal matrix with elements given by

$$\lambda_{ii} = \frac{\Delta t^{p_i}}{p_i!}, \quad i = 1, 2, \dots, m \quad (20)$$

where p_i , $i = 1, 2, \dots, m$, is the lowest order of the derivative of $c_i(\hat{\underline{x}}(t))$ in which any component of $\underline{d}(t)$ first appears due to successive differentiation and substitution for $\hat{\underline{x}}_i(t)$ on the right side. The i^{th} component of $\underline{z}(\hat{\underline{x}}, \Delta t)$ is given by

$$z_i(\hat{\underline{x}}, \Delta t) = \sum_{k=1}^{p_i} \frac{\Delta t^k}{k!} L_f^k(c_i) \quad (21)$$

where $L_f^k(c_i)$ is the k^{th} Lie derivative, defined by

$$\begin{aligned} L_f^k(c_i) &= c_i & \text{for } k = 0 \\ L_f^k(c_i) &= \frac{\partial L_f^{k-1}(c_i)}{\partial \hat{\underline{x}}} \underline{f} & \text{for } k \geq 1 \end{aligned} \quad (22)$$

The i^{th} row of $S(\hat{\underline{x}})$ is given by

$$s_i = \left\{ L_{g_1} \left[L_f^{p_i-1}(c_i) \right], \dots, L_{g_l} \left[L_f^{p_i-1}(c_i) \right] \right\}, \quad i = 1, 2, \dots, m \quad (23)$$

where g_j is the j^{th} column of $G(t)$, and the Lie derivative is defined by

$$L_{g_j} \left[L_f^{p_i-1}(c_i) \right] \equiv \frac{\partial L_f^{p_i-1}(c_i)}{\partial \underline{\hat{x}}} g_j, \quad j = 1, 2, \dots, l \quad (24)$$

Equation (24) is in essence a generalized sensitivity matrix for nonlinear systems. Therefore, given a state estimate at time t_k , then Equation (19) is used to process the measurement at time t_{k+1} to find the $\underline{d}(t_k)$ to be used in $[t_k, t_{k+1}]$ to propagate the state estimate to time t_{k+1} . The weighting matrix W serves to weight the relative importance between the propagated model and measured quantities. This weight may be found by using a measurement error covariance constraint [11].

GPS Attitude Estimation

The nonlinear predictive filter using phase difference measurements minimizes the following cost function

$$J = \frac{1}{2} \left\{ \sum_{i=1}^m \sum_{j=1}^n \sigma_{ij}^{-2} \left[\Delta \tilde{\phi}_{ij}(t_{k+1}) - \underline{b}_i^T(t_{k+1}) A(\underline{\hat{q}}_{k+1}) \underline{s}_j(t_{k+1}) \right]^2 \right\} + \frac{1}{2} \underline{d}^T(t_k) W \underline{d}(t_k) \quad (25)$$

subject to

$$\underline{\dot{q}} = \frac{1}{2} \Omega(\underline{\hat{\omega}}) \underline{\hat{q}} = \frac{1}{2} \Xi(\underline{\hat{q}}) \underline{\hat{\omega}}, \quad \underline{\hat{q}}(t_0) = \underline{q}_0 \quad (26a)$$

$$\underline{\hat{H}} = \underline{N} - \left[J^{-1}(\underline{\hat{H}} - \underline{h}) \right] \times \underline{\hat{H}} + \underline{d}, \quad \underline{\hat{H}}(t_0) = \underline{H}_0 \quad (26b)$$

$$\underline{\hat{\omega}} = J^{-1}(\underline{\hat{H}} - \underline{h}) \quad (26c)$$

where $\underline{\hat{q}}_{k+1} \equiv \underline{\hat{q}}(t_{k+1})$, m is the total number of baselines, n is the total number of available sightlines, and σ_{ij} is the standard deviation of the measurement error noise for the ij^{th} component. The filter may be initialized using a deterministic approach to find the attitude and angular momentum [13]. Since the phase difference measurements ($\Delta \tilde{\phi}_{ij}$) are used as the required tracking trajectories, the model of a single measurement in Equation (15b) is given by

$$c(\underline{\hat{x}}) = \underline{b}_i^T A(\underline{\hat{q}}) \underline{s}_j \quad (27)$$

where

$$\underline{\hat{x}} \equiv \left[\underline{\hat{q}}^T \quad \dots \quad \underline{\hat{H}}^T \right]^T \quad (28)$$

Since \underline{c} depends on $\hat{\underline{q}}$ and not explicitly on $\hat{\underline{H}}$, the lowest order derivative of Equation (27) in which any component of \underline{d} first appears in $\hat{\underline{q}}$ is two, so that $p_i = 2$. Therefore, the Λ and \underline{z} quantities in Equations (20) and (21) are given by

$$\Lambda = \frac{\Delta t^2}{2} I_{3 \times 3} \quad (29a)$$

$$\underline{z} = \Delta t L_{f_{ij}}^1 + \frac{\Delta t^2}{2} L_{f_{ij}}^2 \quad (29b)$$

where the quantities $L_{f_{ij}}^1$ and $L_{f_{ij}}^2$ can be shown to be given

$$L_{f_{ij}}^1 = \underline{b}_i^T [A(\hat{\underline{q}}) \underline{s}_j \times] J^{-1} (\hat{\underline{H}} - \underline{h}) \quad (30)$$

$$L_{f_{ij}}^2 = -\underline{b}_i^T [\hat{\underline{\omega}} \times] [A(\hat{\underline{q}}) \underline{s}_j \times] \hat{\underline{\omega}} + \underline{b}_i^T [A(\hat{\underline{q}}) \underline{s}_j \times] J^{-1} (\underline{N} - [\hat{\underline{\omega}} \times] \hat{\underline{H}}) \quad (31)$$

The S matrix, which is formed using Equation (23) is given by

$$S = \underline{b}_i^T [A(\hat{\underline{q}}) \underline{s}_j \times] J^{-1} \quad (32)$$

The extension to multiple measurement sets is achieved by stacking these measurements, e.g.,

$$\underline{y} = [\Delta\phi_{11} \quad \cdots \quad \Delta\phi_{m1} \mid \Delta\phi_{12} \quad \cdots \quad \Delta\phi_{m2} \mid \cdots \mid \Delta\phi_{1n} \quad \cdots \quad \Delta\phi_{mn}]^T \quad (33)$$

Similar stacking of the quantities \underline{z} and S are used in the predictive filter. Other state variables, such the addition to line biases, can also be added easily [10]. The major advantage of the predictive filter over the traditional Kalman filter is that the torque modeling error (\underline{d}) is determined as part of the predictive filter's solution, whereas the state vector in the Kalman must be augmented in order to estimate for unmodeled torque disturbances and errors [10].

Another case involves using quaternion measurements given from a deterministically found attitude using GPS phase difference measurements ([1], [13], [14]). If the determined quaternions are used in the predictive filter, then the following cost function is minimized

$$J = \frac{1}{2} \{ \tilde{\underline{q}}(t_{k+1}) - \hat{\underline{q}}(t_{k+1}) \}^T R^{-1} \{ \tilde{\underline{q}}(t_{k+1}) - \hat{\underline{q}}(t_{k+1}) \} + \frac{1}{2} \underline{d}^T(t_k) W \underline{d}(t_k) \quad (34)$$

where $\tilde{\underline{q}}$ denotes the determined quaternion using the GPS phase difference measurements. For this case, the quantities Λ , \underline{z} , and S can be shown to be given by [15]

$$\Lambda = \frac{\Delta t^2}{2} I_{4 \times 4} \quad (35a)$$

$$\underline{z} = \frac{\Delta t}{2} \Xi(\hat{q}) \hat{\omega} - \frac{\Delta t^2}{8} \left\{ (\hat{\omega}^T \hat{\omega}) \hat{q} + 2 \Xi(\hat{q}) J^{-1} ([\hat{\omega} \times] \underline{H} - \underline{N}) \right\} \quad (35b)$$

$$S = \frac{1}{2} \Xi(\hat{q}) J^{-1} \quad (35c)$$

The case simplifies the calculations required in the filter; however, the approach using the cost function in Equation (25) (i.e., using the phase difference measurements) can in theory determine the attitude using one baseline, while deterministic methods fail in this case. Also, optimal determination of the quaternion from phase measurements requires a computationally expensive gradient search, and more efficient methods can be suboptimal [13]. The computation simplifies if the measurement error covariance (R) is represented as a scalar (r) times the identity matrix, due to the fact that $\Xi^T(\hat{q}) \hat{q} = \underline{0}$ and $\Xi^T(\hat{q}) \Xi(\hat{q}) = I_{3 \times 3}$. This leads to

$$\underline{d}(t_k) = \left[J^{-2} + \frac{8r}{\Delta t^4} W \right]^{-1} J^{-1} \left\{ \frac{4}{\Delta t^2} \Xi^T(\hat{q}_k) \tilde{q}_{k+1} + J^{-1} ([\hat{\omega}_k \times] \hat{H}_k - \underline{N}_k) - \frac{2}{\Delta t} \hat{\omega}_k \right\} \quad (36)$$

Also, note that the inverse in Equation (36) has to be computed only once, which greatly simplifies the computational load. Also, if the weighting matrix W is set to zero, then Equation (36) invokes a feedback linearization of the dynamics model in Equation (26b) [15].

Attitude Estimation of REX-II

In this section, the predictive filter is used to estimate the attitude of the REX-II spacecraft. A drawing of the REX-II spacecraft is shown in Figure 2 (for a more complete description of the spacecraft see Ref. [6]). The spacecraft is passively stabilized using a 6 meter boom with gravity gradient torques and magnetic hysteresis rods for damping. REX-II is additionally actively controlled by electromagnetic coils and a pitch-axis reaction wheel, which provides a momentum bias. The vehicle attitude is expressed as a 3-2-1 yaw-pitch-roll Euler sequence [16] from the locally level orientation, as shown in Figure 3 [6]. Also, the spacecraft contains a three-axis magnetometer (TAM), from which magnetic field measurements are simultaneously available with the GPS deterministic attitude solutions.

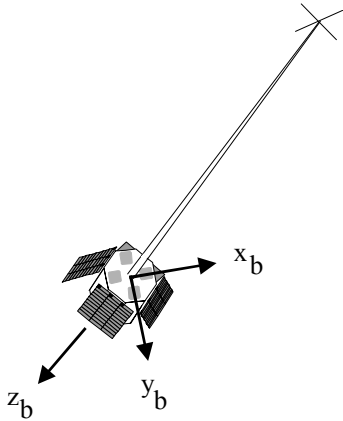


Figure 2 REX-II On-Orbit Configuration

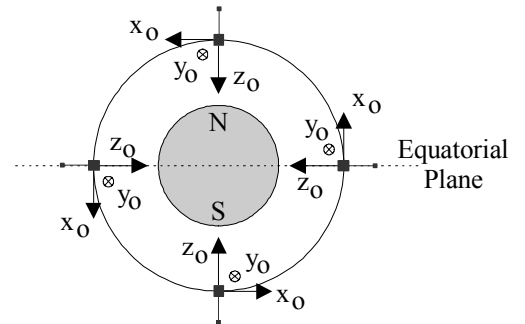


Figure 3 Locally Level Reference Attitude

The four GPS patch antennas were mounted on the top surface of the spacecraft main body in a coplanar, aligned configuration, as shown in Figure 4 [6]. All four antennas were keyed in the same direction to provide antenna phase center repeatability. The separation is 0.67 meters along the diagonal, and the gravity gradient boom extends out of the center of the main body. The M-2 and 1-3 axes were mechanically aligned to within 0.1 degrees of the spacecraft x and y body axes, respectively.

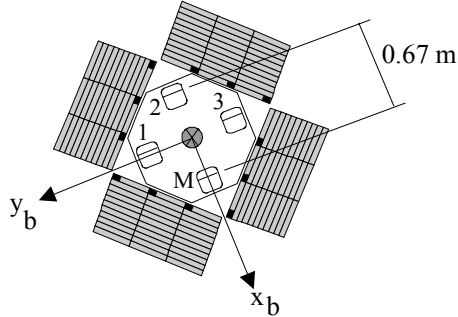


Figure 4 Antenna Placement on Spacecraft

Attitude Estimation

In this section, the predictive filter is used to estimate the attitude accuracy of the REX-II spacecraft. Since the REX-II telemetry system downloads the attitude quaternion from a deterministically found attitude, Equation (36) involving quaternion measurements is used. Also, the problem is complicated by the fact that the telemetry system is constrained to operate with low bandwidth over short ground passes (10 minutes), resulting in short time spans with full data, or very low sampling rates that cover longer times spans. Despite this complication, the GPS solution performance may still be estimated in comparison with an independent TAM-only attitude solution. Furthermore, the TAM/GPS tandem is expected to be common on future low-Earth orbit spacecraft, so it is also worthwhile to consider the potential for combining these measurements into a single, more accurate estimate of spacecraft attitude.

In order to provide an accurate (as possible) analytical model, active control laws (magnetic moment torques and the momentum bias wheel) and passive control laws (gravity gradient torques and magnetic hysteresis rods) were simulated. However, simulations showed that the fit of the data to experimental results is much better with the effect of the hysteresis rods omitted, than with this effect included [6]. This may indicate that the rods were compromised during spacecraft assembly, launch, or deployment such that the material is no longer effective.

Once a fairly accurate representation of the response of the analytical model was obtained, the next step was to design an extended Kalman filter using this model and TAM measurements only. After many attempts at tuning the filter, a reasonably estimated attitude could not be found. This may be due to the fact that data from a short time span (less than 1/3 of the orbit period) is only available at one time, and the Kalman filter may require more data to converge. Although the authors do not claim that a solution may not exist using a Kalman filter, more reasonable attitude solutions were achieved using the predictive filter approach shown in Ref. [17]. Also, the predictive filter using TAM-only measurements estimated the attitude using a dynamics model with the wheel torque only.

Since REX-II did not fly with gyros, and due to the data set limitations (either too short for filter convergence, or sampled very far apart), an accurate attitude reference that may be used as “truth” to the benchmark GPS accuracy is not possible. Other effects, such as uncertainties in the magnetic

field model, and potential magnetic activity near the TAM, further complicate this issue. However, at worst, a sanity check of the GPS attitude behavior is still feasible, since attitude accuracy between 0.5 to 2 degrees is possible using a dynamics model and TAM measurements only as shown in Ref. [18].

The sampling rate is 20 seconds for the short-time span set, which lasted for 25 minutes. The long-time span data was also considered. This lasted for 24 hours, but was sampled at 6 minute intervals. A study was performed on the long span of data to test the validity of using this data for attitude estimation. This involved using a spline fit interpolation scheme to bridge the data gap. Unfortunately, a coning motion is evident along the y axis at a higher frequency than the 6 minute sample interval reveals, shown by Figure 5. The circles show 6 minute sampling, the dotted lines show the spline fit, and the solid lines show the measurements from the short span TAM measurements. The stability of the z and x directions is apparent, since the spline fit data virtually lie on the measurement data. However, the high frequency activity is clear in the y axis, as well as the 0.004 gauss quantization.

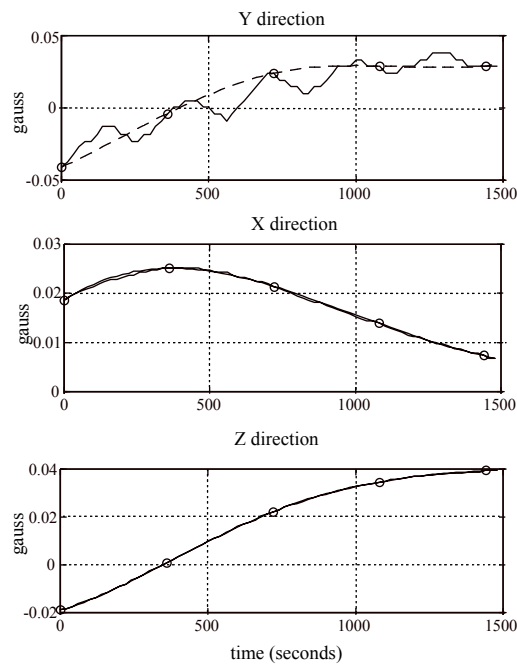


Figure 5 TAM Data for Short Span (solid), Long Span (circles), and Spline Fit (dashed)

Figure 6 shows the magnitude difference between the GPS attitude point (deterministic) solutions and the TAM-only predictive filter solutions. Although only a 25 minute (1/3 orbit) span is available, the filter was able to converge to a solution. A Kalman filter typically requires a full orbit to converge [18]. Therefore, the predictive filter provided convergence at a faster rate than an equivalent Kalman filter approach. The error in the reference geomagnetic field model, used in the TAM-only attitude computation, could be as much as 2 degrees, so that the difference in the two attitudes appears to be converging to a value within that tolerance. Further, this attitude difference seems to be smaller than for the simulated data [6]. The second plot in Figure 6 shows, for the same data span, the magnetic field measurement residual. Here, the magnetometer measurement was transformed into inertial space by both the TAM-only filtered attitude (the solid line) and GPS attitude point solutions (the dashed line). These values are then compared to the reference geomagnetic field model; where the three-axis differences are presented in Gauss. The final magnetometer residual (0.013 gauss) is equivalent to about a 1.8 degree angular residual, and the GPS point solution at that time is about 4.8 degrees. These residuals also appear to be within values predicted by the simulations [6].

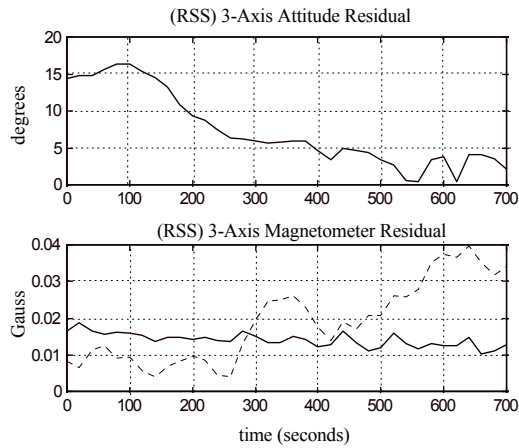


Figure 6 Attitude and TAM Error Residuals (see text for explanation)

The final case involves the blending the GPS measurements and TAM measurements using the predictive filter for a combined attitude estimate. For this case, it was determined that the measurement error sources should be nearly equally weighted to achieve the lowest residuals between the combined estimated attitude and the TAM-only estimated attitude. A plot of the magnitude residuals using this approach is shown in Figure 7. The combined predictive filter attitude seems to be converging to a residual of about 2 degrees. Although this value cannot be known accurately for this system, the methodology of the combined predictive filter approach seems to provide a reasonable method for attitude estimation.

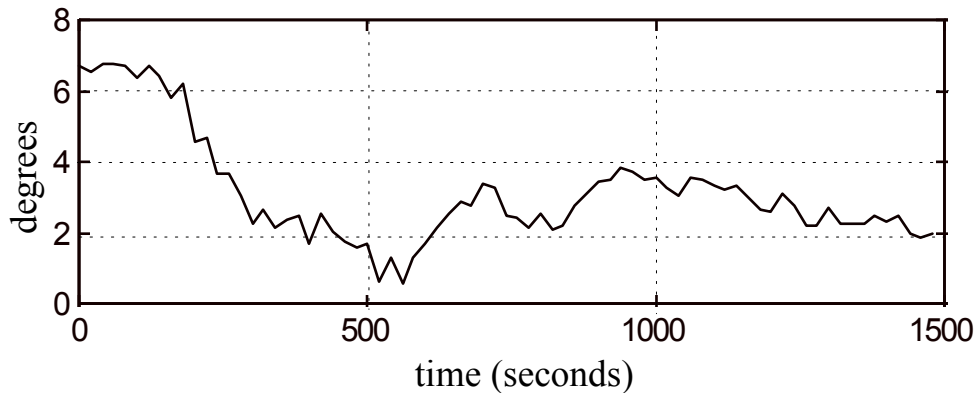


Figure 7 GPS/TAM and TAM Attitude Error Residual

Conclusions

In this paper, a predictive filter scheme was presented for attitude estimation using GPS sensor observations. This algorithm was specifically developed for spacecraft which lack angular rate sensing equipment. The new algorithm was applied to the REX-II spacecraft to investigate attitude accuracy. It was shown that the predictive filter was able to converge in less than 25 minutes (1/3 of an orbit), and was able to estimate the attitude using wheel inputs only in the dynamic model. This has clear advantages over a Kalman filter, which typically requires a compromise between convergence rate and steady-state performance, and usually requires more extensive modeling of control and disturbance torques in the dynamics model. The magnetometer validation of the GPS attitude solutions agrees to within the measurement accuracy of the magnetic field attitude determination method (about 2 degrees). Also, GPS solutions and magnetic field measurements were combined into a single estimate for the REX-II spacecraft attitude. This combined sensor output estimator is considered to be more practical for sub-degree controller performance because it provides an acceptable attitude measurement even during periods of GPS attitude sensor outage, which has been shown to occur routinely with current GPS receiver hardware during normal spacecraft operations. GPS attitude accuracy of better than 1 degree is expected per axis, but cannot be reliably proven with this sensor complement.

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