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A COMPARATIVE STUDY OF SLIDING MODE CONTROL AND TIME-OPTIMAL CONTROL

Jongrae Kim*, John L. Crassidis†

ABSTRACT

A comparative study of sliding mode control and time-optimal control for spacecraft attitude control using thrusters is presented. The on-off type thrusters are assumed to be attitude control actuators. Application of time-optimal control theory to the kinematic and dynamic equations of motion leads to a nonlinear two-point boundary value problem. The solution can be computed by iterative numerical methods, but typically, the computing method is not compatible with real-time computing constraints. In this paper, sliding mode control is used for the input of PWPF (Pulse Width and Pulse Frequency) modulator thrusters. The settling times are minimized by tuning the gains of sliding mode control. The sliding mode control is derived using modified Rodrigues parameters. We also derive the upper bound of sliding function for inertia uncertainty and external disturbances. Disturbance accommodating sliding mode control with PWPF is derived to minimize the settling time with inertia uncertainty and external disturbance. Simulation results show that the settling times are shorter than the ones of the sliding mode control with PWPF.

INTRODUCTION

Spacecraft attitude control for large-angle slewing maneuvers poses a difficult problem, including: the nonlinear characteristics of the governing equation, modeling uncertainty and unexpected external disturbances. Usually, for rapid and coarse attitude maneuvers, on-off type thrusters have been used. Control algorithms can be divided into open-loop systems and closed-loop systems. Application of time-optimal control theory to the kinematic and dynamic equations of motions leads to a nonlinear two-point boundary value problem. Time-optimal control for spacecraft attitude maneuvers is different from the eigenaxis rotation.[1] The solution can be computed by iterative numerical methods, but the computing method is not compatible with real-time computing constraint. Also, it is sensitive with respect to modeling errors and external disturbances.

Sliding mode (variable structure) control provides robustness with respect to modeling errors and is an effective method for handling the nonlinear characteristics for attitude control. Vadali presented an optimal sliding manifold using error quaternions.[2] In his paper, two types of actuators, i.e., thrusters and reaction wheels, are used for the spacecraft maneuver. In this paper, the sliding mode control using modified Rodrigues parameters are combined with a PWPF (Pulse Width Pulse Frequency) modulator.[3] The PWPF modulator drives a thruster valve with an on-off pulse sequence having a nearly linear duty cycle with the input amplitude. The settling time is minimized by tuning the gains in the sliding mode control using standard optimization techniques.[4]

One of the drawbacks of sliding mode control is the chattering problem due to disturbance and modeling imprecision. For spacecraft attitude control, chattering may excite the higher frequencies of the spacecraft. Chattering can be settled by smoothing the control input using boundary layer or bandwidth-limited sliding mode control, which was presented by Dwyer and Kim.[5] However, a globally suitable boundary layer thickness cannot be easily determined. Moreover, for spacecraft attitude control it may be difficult to predict the external disturbances acting on body. When bounded unmodeled external torques are added, the closed-loop system is no longer globally asymptotically stable since a steady-state error is present. The error can be minimized by increasing the correction control gain or decreasing the thickness of the boundary layer in sliding mode control. In this paper we de-

*Graduate Student, Department of Aerospace Engineering, Texas A&M University, College Station, Texas, 77843, Student Member AIAA

†Assistant Professor, Department of Aerospace Engineering, Texas A&M University, College Station, Texas, 77843, Senior Member AIAA

rive this relation using a Lyapunov function. But for limited actuator capability the maximum correction control gain and the minimum thickness of boundary layer being allowed may be restricted. Though the steady-state errors are usually small, in a high-precision attitude pointing or tracking systems, these errors may not tolerable for satisfying a mission requirement.

In this paper, we adopt disturbance accommodating control to minimize steady-state errors in sliding mode control. The disturbance accommodating control concept was first proposed by Johnson.[6,7] External disturbances $w(t)$ are assumed to satisfy $d^{m+1}w(t)/dt^{m+1} = 0$ differential equation where the external disturbances are represented as mth-degree polynomials in time t with unknown coefficients.[7] Design procedures and existence of the disturbance observer are presented in [8] and [9]. In these papers, a disturbance accommodating observer is combined with a control method that provides linear behaviors in the responses of the systems. Advantages of using disturbance accommodating observer include the following: 1) it is linear, and 2) it also compensates the error due to modeling uncertainty.

Combining sliding mode control with a disturbance accommodating observer (i.e., Disturbance Accommodating Sliding Mode Control) was presented by Kim, and was applied to a robot manipulator for reducing the upper bound of bandwidth of sliding mode control.[11] In this paper sliding mode control based on modified Rodrigues parameters is adopted for spacecraft attitude control. Also, a disturbance accommodating observer is combined with sliding mode control in order to reduce the settling time with inertia uncertainty and external disturbances.

The organization of this paper proceeds as follows. First, a brief summary of the kinematics and dynamics of a spacecraft is presented. Then, a brief overview of the sliding mode control based on modified Rodrigues parameters and PWPF modulator is shown. Next, a robust analysis of the sliding mode control with respect to modeling errors and external disturbances is accomplished using a Lyapunov function. A disturbance accommodating observer is derived for reducing the settling time. Also, sliding mode control and disturbance accommodating observer are combined. Finally, we compare the settling time with the ones of time-optimal solution and demonstrate the performance against modeling errors and external disturbances.

PROBLEM FORMULATION

In this section, a brief review of the kinematic equations of motion using modified Rodrigues parameters, the rigid body dynamics, sliding mode control based on the kinematics, and PWPF modulator is shown.

Attitude Kinematics and Dynamics

The modified Rodrigues parameters are defined by[13]

$$\mathbf{p} \equiv \hat{\mathbf{n}} \tan(\theta/4) \tag{1}$$

where $\mathbf{p}$ is a 3×1 vector, $\hat{\mathbf{n}}$ is a unit vector corresponding to the axis of rotation and θ is the angle of rotation. The kinematic equations of spacecraft attitude motion described in modified Rodrigues parameters are derived by using the spacecraft's angular velocity ($\boldsymbol{\omega}$), given by[13]

$$\dot{\mathbf{p}} = \frac{1}{4}\left\{\left(1 - \mathbf{p}^T\mathbf{p}\right) I_{3\times3} + 2\left[\mathbf{p}\times\right] + 2\,\mathbf{p}\mathbf{p}^T\right\}\boldsymbol{\omega} \tag{2}$$

where $\mathbf{p}^T$ is the transpose of $\mathbf{p}$, $I_{3\times3}$ is a 3×3 identity matrix, and $[\mathbf{p}\times]$ is a 3×3 cross product matrix defined by

$$[\mathbf{p}\times] = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix} \tag{3}$$

The dynamic equation of motion for a rigid body with external disturbance ($\mathbf{w}$) is given by Euler's equation, defined by

$$\dot{\boldsymbol{\omega}} = J^{-1}\left[J\boldsymbol{\omega}\times\right]\boldsymbol{\omega} + J^{-1}\mathbf{u} + J^{-1}\mathbf{w} \tag{4}$$

where, J is the spacecraft's inertia (3×3) matrix, J^{-1} is the inverse matrix of J, and $\mathbf{u}$ is the control input torque (3×1) vector.

Sliding Mode Control

In this paper it is assumed that measurements of both the spacecraft attitude and angular rate are available and the dynamics of the actuator is neglected. The nonlinear model for spacecraft motion is summarized by[13]

$$\dot{\mathbf{p}} = F(\mathbf{p})\,\boldsymbol{\omega} \tag{5}$$

$$\dot{\boldsymbol{\omega}} = \mathbf{f}(\boldsymbol{\omega}) + J^{-1}\mathbf{u} + J^{-1}\mathbf{w} \tag{6}$$

where

$$\begin{aligned} F(\mathbf{p}) &\equiv 1/4\left\{\left(1 - \mathbf{p}^T\mathbf{p}\right) I_{3\times3} + 2\left[\mathbf{p}\times\right] + 2\,\mathbf{p}\mathbf{p}^T\right\} \\ \mathbf{f}(\boldsymbol{\omega}) &\equiv J^{-1}\left[J\boldsymbol{\omega}\times\right]\boldsymbol{\omega} \end{aligned}$$

Sliding mode control introduces velocity vector fields directed toward the sliding surface or manifold ($\mathbf{s} = 0$) in its immediate vicinity, where $\mathbf{s}$ is given by[12]

$$\mathbf{s} \equiv \boldsymbol{\omega} - \mathbf{m}(\mathbf{p}) \qquad (7)$$

The quantity $\mathbf{m}(\mathbf{p})$ is defined using a desired vector field from the kinematic equation, given by[12]

$$\mathbf{m}(\mathbf{p}) = F^{-1}(\mathbf{p})\mathbf{d}(\mathbf{p}) \qquad (8)$$

The quantity $\mathbf{d}(\mathbf{p})$ is formed by allowing a linear behavior in the sliding motion, given by[13]

$$\mathbf{d}(\mathbf{p}) = \Lambda\,(\mathbf{p} - \mathbf{p}_d) \qquad (9)$$

where $\mathbf{p}_d$ is the desired reference trajectory and Λ is a diagonal matrix with negative elements. The input by sliding mode control is divided into two parts. The first is the equivalent control $\mathbf{u}_{eq}$ for satisfying the ideal sliding mode conditions (i.e., invariant conditions). The second is the correction control $\mathbf{u}_{cr}$ for satisfying the sliding mode existence conditions.[12] As a result, the control input is given by

$$\mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_{cr} \qquad (10)$$

where

$$\mathbf{u}_{eq} = -J\left\{\mathbf{f}(\boldsymbol{\omega}) - \frac{\partial \mathbf{m}}{\partial \mathbf{p}}F(\mathbf{p})\left[\mathbf{m}(\mathbf{p}) + \mathbf{s}\right]\right\} \qquad (11)$$

$$\mathbf{u}_{cr} = -JKsat(\mathbf{s}, \epsilon) \qquad (12)$$

where K is a 3×3 positive definite diagonal matrix. The saturation function is used to minimize chattering in the control torques. The function is defined by

$$sat(s_i, \epsilon) \equiv \begin{cases} 1 & \text{if } s_i > \epsilon \\ s_i/\epsilon & \text{if } |s_i| \le \epsilon \\ -1 & \text{if } s_i < \epsilon \end{cases} \qquad (13)$$

The detailed descriptions of the quantities $\mathbf{m}(\mathbf{p})$ and $\partial\mathbf{m}/\partial\mathbf{p}$ for the regulation and the tracking problems can be found in [13].

PWPF Modulator

PWPF modulator is composed of a first-order lag filter and a Schmitt trigger inside a feedback loop as shown in Fig. 1. In contrast to the Schmitt trigger, the static characteristics, i.e., thruster pulse width, off-time, frequency and duty cycle, etc., of the PWPF modulator are independent of the spacecraft inertia.[3]

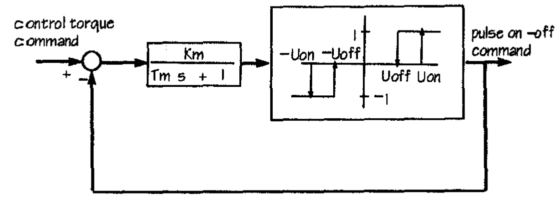

Fig. 1 PWPF modulator

CONTROL DESIGN

In this section a robust analysis of the sliding mode control with respect to modeling errors and external disturbances is accomplished using a Lyapunov function. A disturbance accommodating observer is also derived in order to reduce the effect of inertia uncertainty and external disturbances. Finally, sliding mode control and disturbance accommodating observer are combined.

Robust Analysis of Sliding Mode Control

We use the following candidate Lyapunov function V to study global stability of the motion by sliding mode control[14]

$$V = \frac{1}{2}\mathbf{s}^T J \mathbf{s} \qquad (14)$$

We define the bounded modeling error as follows:

$$J = \hat{J} + \Delta J \qquad (15)$$

$$J^{-1} = \hat{J}^{-1} + \Delta J^{-1} \qquad (16)$$

In general, the inverse of $\hat{J}$ and ΔJ are not equal to $\hat{J}^{-1}$ and ΔJ^{-1} respectively. The first time derivative of the candidate Lyapunov function with the control input reduces to

$$\dot{V} = -\mathbf{s}^T\Big[J\Delta J^{-1}\hat{J}\frac{\partial \mathbf{m}}{\partial \mathbf{p}}F(\mathbf{p})(\mathbf{s} + \mathbf{m}) + \hat{J}Ksat(\mathbf{s}, \epsilon) + J^{-1}\mathbf{w}\Big] \qquad (17)$$

In the above equation, we neglect the nonlinear part of dynamic equation. Substituting the control torque into the first time derivative of sliding function leads to the following

$$\dot{\mathbf{s}} = \Delta J^{-1}\hat{J}\frac{\partial \mathbf{m}}{\partial \mathbf{p}}F(\mathbf{p})(\mathbf{s} + \mathbf{m}) - J^{-1}\hat{J}Ksat(\mathbf{s}, \epsilon) + J^{-1}\mathbf{w} \qquad (18)$$

We assume that the thickness of boundary layer ϵ is sufficiently small and the correction control gain

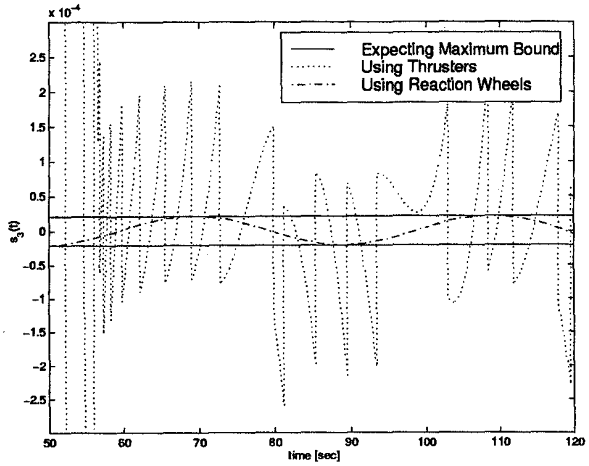

Fig. 2 The upper and lower bound of sliding function

K is sufficiently large to keep the time derivative of Lyapunov function negative-definite with modeling errors and bounded external disturbances in the region of the outer boundary layer. After neglecting the relatively small term in the boundary layer the dynamics of sliding function is given by

$$\dot{\mathbf{s}} = -\frac{1}{\epsilon}J^{-1}\hat{J}K\mathbf{s} + J^{-1}\mathbf{w} \tag{19}$$

If the sliding function $\mathbf{s}$ settles to a finite constant $\mathbf{s}_{ss}$, the steady-state value satisfies the following inequality

$$|\mathbf{s}_{ss}| \leq \epsilon[\hat{J}K]^{-1}|\mathbf{w}|_{\mathbf{max}} \tag{20}$$

As shown in Fig. 2, when we use reaction wheels as actuators, the sliding function trajectory is bounded in the expected maximum bound. However, in the case of thrusters, due to the restricted minimum pulse width the trajectory oscillates up and down from the expected bound.

Disturbance Accommodating Observer

The uncertainty associated with some internal and external disturbances $\mathbf{w}(t)$ is represented by a semideterministic waveform-model description of the generalized spline-function type, given by[10]

$$\mathbf{w}(t) = \mathbf{c_1}f_1(t) + \mathbf{c_2}f_2(t) + \cdots + \mathbf{c_m}f_m(t) \tag{21}$$

where the basis functions $f_1(t),\ f_2(t),\ \cdots\ f_m(t)$ are completely known and the constant weighting coefficient vectors $\mathbf{c_1},\ \mathbf{c_2},\ \cdots\ \mathbf{c_m}$ are totally unknown and may jump in value from time to time. Without loss of generality, it is further assumed that the basis functions $f_i(t)$ satisfy a linear differential equation. As a consequence, there exists a linear dynamical "state model" representation as follows:[10]

$$\mathbf{w}(t) = H(t)\mathbf{z} \tag{22}$$

$$\dot{\mathbf{z}} = D(t)\mathbf{z} + \boldsymbol{\sigma}(t) \tag{23}$$

where $H(t),\ D(t)$ are completely known and $\boldsymbol{\sigma}(t)$ is a vector of impulse sequences representing jumps in the $\mathbf{c_i}$ which are sparse but otherwise totally unknown. The waveform and state models have been successfully used to represent plant model errors associated with the following items:[10]

- coulomb and other complex forms of nonlinear damping
- uncertain external input disturbances
- plant parameter model errors
- coupling effects in reduced-order state models

The basis functions can be chosen as power series in time t or as orthogonal polynomials commonly used in approximation theory.[10] The design procedure and the existence problem of the appropriate observer with the stabilizing gain was shown in [9].

Disturbance Accommodating Sliding Mode Control

In this paper we divide the control input into the equivalent control input $\mathbf{u}_{eq}$ and the correction control input $\mathbf{u}_{cr}$ of the sliding mode control and the disturbance accommodating control input $\mathbf{u}_{dac}$ for canceling the effects of external disturbances:[10,11]

$$\mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_{cr} + \mathbf{u}_{dac} \tag{24}$$

After applying the control input to the dynamics of the sliding function, the dynamics and the disturbance model can be written in the following state-space form:[10,11]

$$\begin{aligned} \dot{\mathbf{s}} &= J^{-1}\mathbf{u}_{cr} + J^{-1}\mathbf{u}_{dac} + J^{-1}\mathbf{w} && (25)\\ \dot{\mathbf{z}} &= D(t)\mathbf{z} + \boldsymbol{\sigma}(t) && (26)\\ \mathbf{w} &= H(t)\mathbf{z} && (27) \end{aligned}$$

The appropriate disturbance accommodating observer is given by[9]

$$\begin{aligned} \dot{\hat{\mathbf{z}}} &= D(t)\hat{\mathbf{z}} - K_0(\mathbf{z} - \hat{\mathbf{z}}) && (28)\\ \hat{\mathbf{w}} &= H(t)\hat{\mathbf{z}} && (29) \end{aligned}$$

where K_0 is the observer gain (9×9) matrix which provides sufficient time constants in the observer.

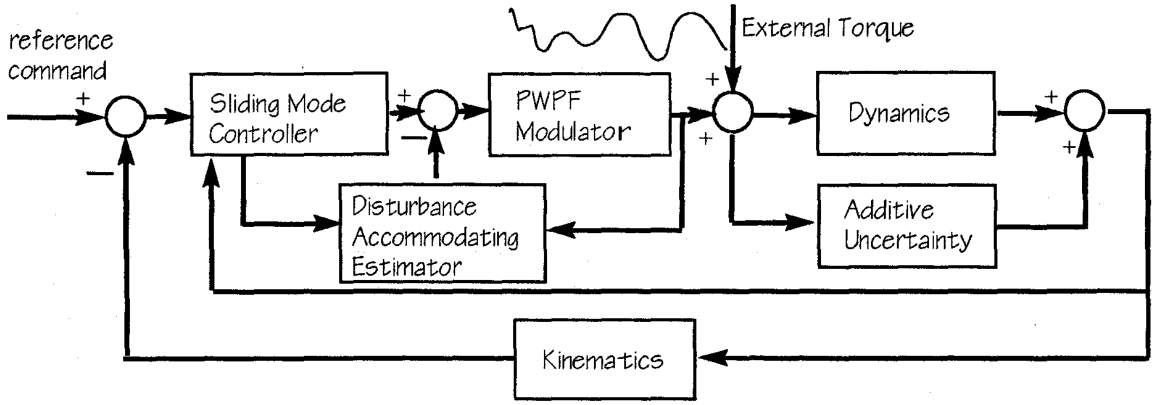

Fig. 3 System Block Diagram

We adopt the three basis functions as 1, t, t^2 for each body axis (i.e., $i = 1,\ 2,\ 3$)

$$w_i(t) = c_1 + c_2 t + c_3 t^2 \tag{30}$$

We assume that the time derivative of the jerk of external disturbance is zero (i.e., $d^3 w_i(t)/dt^3 = 0$), so that H_i, D_i are given by

$$H_i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{31}$$

$$D_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{32}$$

All the matrices in the observer are constant, however, the observer in Eq. (28) cannot be directly implemented due to the unmeasurable state $\mathbf{z}$. Define a new state variable $\mathbf{Q}$ as follows:[11]

$$\mathbf{Q} = \hat{\mathbf{z}} - K_1 \mathbf{s} \tag{33}$$

where K_1 is a gain matrix (9×3). The gain K_1 can be tuned to satisfy the following condition:[11]

$$K_0 + K_1 J^{-1} H = 0 \tag{34}$$

Finally, the modified observer composed by the measurable or known states is derived as follows:[15]

$$\begin{aligned}\dot{\mathbf{Q}} &= (D + K_0)\mathbf{Q} + (D + K_0)K_1\mathbf{s} \\ &- K_1 J^{-1}(\mathbf{u}_{cr} + \mathbf{u}_{dac})\end{aligned} \tag{35}$$

where the initial condition is given by $\mathbf{Q}(0) = -K_1\mathbf{s}(0)$. Then, the estimation error dynamics become[15]

$$\Delta\dot{\mathbf{Q}} - (D + K_0)\,\Delta\mathbf{Q} = -\sigma(t) \tag{36}$$

where

$$\Delta\mathbf{Q} = (\hat{\mathbf{z}} - K_1\mathbf{s}) - (\mathbf{z} - K_1\mathbf{s}) \tag{37}$$

If the gain K_0 is large enough so that the error dynamics is stable and converges fast, then the tracking error offset is reduced. The designed observer is linear and it can be easily implemented in digital software. One of drawbacks of the observer is that the sensor noise is amplified by the gain at the output of the observer. In this case we cannot use the reduced observer form, and have to implement a observer to estimate the state $\mathbf{s}$.

A brief description of the control and system is shown in Fig. 3. The estimated states $\hat{\mathbf{z}}$, $\hat{\mathbf{w}}$ and $\mathbf{u}_{dac}$ are calculated by the following relation:[10,11]

$$\hat{\mathbf{z}} = \mathbf{Q} + K_1\mathbf{s} \tag{38}$$

$$\hat{\mathbf{w}} = H\hat{\mathbf{z}} \tag{39}$$

$$\mathbf{u}_{dac} = -\hat{\mathbf{w}} \tag{40}$$

SIMULATION

In this section, first, reaction wheels are applied to show the control errors using sliding mode control and disturbance accommodating sliding mode control. Second, we will compare the results of sliding mode control and disturbance accommodating sliding mode control using thrusters with time-optimal solutions. Second, for 180 degrees yaw maneuver, we will compare the settling times and trajectories for each control methods, i.e., time-optimal control, sliding mode control with PWPF and disturbance accommodating sliding mode control with PWPF. The initial conditions for the angular velocity are set to zero. The boundary layer thickness ϵ in the saturation controller is set to 0.01. The K_m, T_m of the lag filter in PWPF are given by 7.46 and 1.33 respectively. Also, U_{on} is 0.45 and U_{off} is 0.25. The simulations are performed by Runge-Kutta 5th method in simulink in MATLAB with a maximum step size of 0.1 sec, minimum step size of 0.0001 sec and a tolerance 1.0×10^{-6}.

Sliding mode control and Distrubance Accommodating Sliding Mode Control

The inertia matrix of the simulated spacecraft and the estimated inertia matrix are as follows respectively:

$$J = diag\left[\ 125.4 \quad 108.3 \quad 131.1\ \right] [kg\ m^2] \tag{41}$$

$$\hat{J} = diag\left[\ 114 \quad 114 \quad 114\ \right] [kg\ m^2] \tag{42}$$

The inertia uncertainties for each body axis are about -9.1%, 5.26% and -13.04%. The external disturbances applied to each body axis are set to

$$\begin{aligned} w_1 &= 0.001\sin(t) \ \ \text{[N-m]} & (43)\\ w_2 &= -0.001\sin(t) \ \ \text{[N-m]} & (44)\\ w_3 &= 0.001\sin(t) \ \ \text{[N-m]} & (45) \end{aligned}$$

The observer gain K_{0_i} for each body axis (i.e., $i =$ 1, 2, 3) is calculated using a pole-placement method as the following:

$$K_{0_i} = \begin{bmatrix} -30.0 & 0 & 0 \\ -300.0 & 0 & 0 \\ -1000.0 & 0 & 0 \end{bmatrix} \tag{46}$$

The gains λ and K are given by 0.3854 and -0.0421 respectively. Reaction wheels are used as the control actuators. We apply bounded inertia uncertainties and external disturbances. In the figure, DAC stands for disturbance accommodating control.

As shown in Fig. 4, the trajectory of disturbance accommmodating sliding mode control approaches zero. In the case of sliding mode control, the trajectory oscillates due to the uncertainties and the external disturbances. The estimated disturbance for yaw axis is shown in Fig. 5. About up to 35 seconds, the estimated value is large due to the inertia uncertainties and relatively large angular rate at the initial maneuver.

Time-optimal control

The inertia matrix is assumed to be given by identity matrix with no uncertainty in the inertia matrix and with no external disturbances. The gains, λ and K are selected to minimize the settling time using the fmins function in optimization toolbox in MATLAB. The optimized results are given by Table. 1, where SMC stands for sliding mode control and TOC stands for time-optimal control. The rotation maneuvers are all about the yaw axis. The settling times are larger than 40% with respect to the ones of time-optimal solution. As shown in the Table. 1 as the rotation angle becomes larger, the relative difference is smaller.

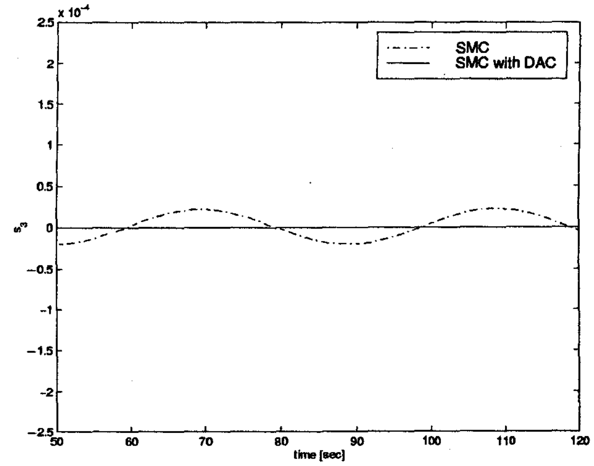

Fig. 4 Trajectories of s_3

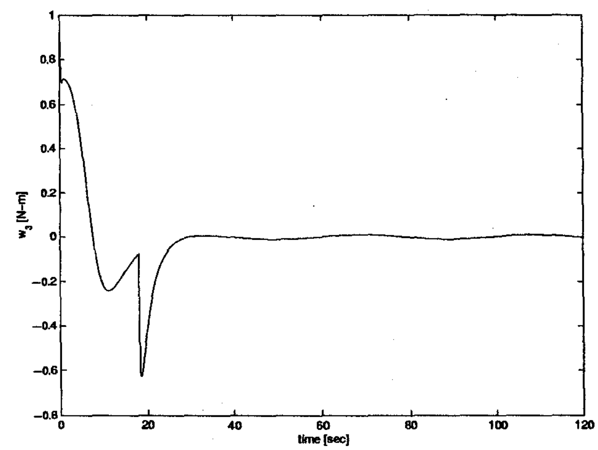

Fig. 5 Estimated Disturbance

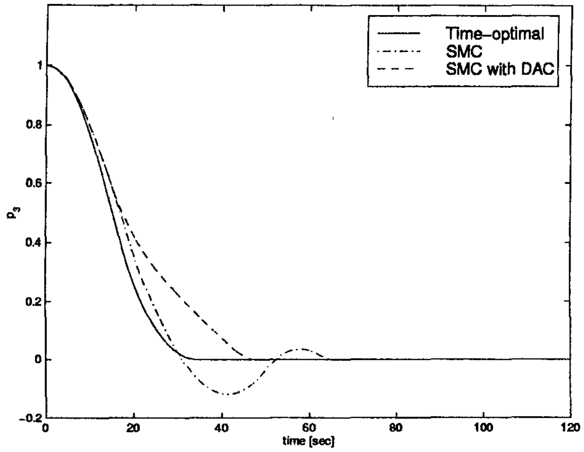

Fig. 6 Case A: Trajectories of the p_3

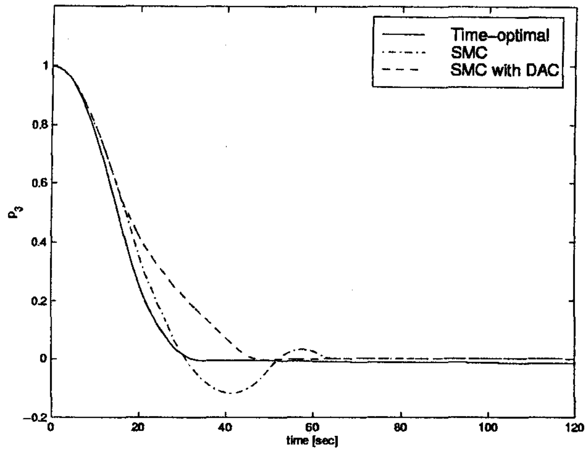

Fig. 7 Case B: Trajectories of the p_3

Table. 1 Optimal gains and settling times

Degree	λ	k	SMC (sec)	TOC[1] (sec)
180°	-2.2437	1.3960	4.7478	3.2431
135°	-2.3287	1.5527	4.2822	2.8845
90°	-2.1917	1.4262	3.8747	2.4211
73°	-2.2396	1.4529	3.6252	2.2024
72°	-2.2396	1.4166	3.6020	2.1885
45°	-2.2812	1.4194	2.9598	1.7499
10°	-2.3240	1.4017	2.3400	0.8334

([1]Ref. 1)

Robustness

In this section, we compare the settling times and trajectories for the following control algorithms:

- Time-optimal control.
- Sliding mode control with PWPF.
- Disturbance accommodating sliding mode control with PWPF.

Simulation cases are also given by

- Case A: Without inertia uncertainty and external disturbances.
- Case B: Without inertia uncertainty and with external disturbances.
- Case C: With inertia uncertainty and without external disturbances.
- Case D: With inertia uncertainty and external disturbances.

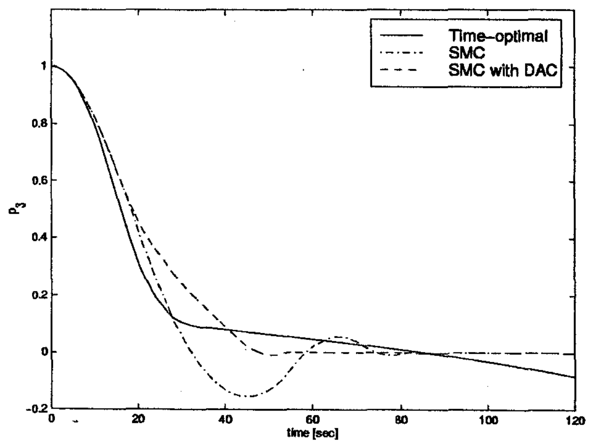

Fig. 8 Case C: Trajectories of the p_3

Fig. 6 shows the trajectories of p_3 for each control method. In this case, the settling time of disturbance accommodating sliding mode control is faster than the one for sliding mode control. As shown in Fig. 7, the settling time of disturbance accommodating sliding mode control for the Case B is also shorter than sliding mode control. The trajectory of time-optimal control slowly diverges with a value of about -0.02 at 120 seconds. This is due to the fact that open-loop control is not robust. In the Case C, as shown in Fig. 8, the settling time of disturbance accommodating control is 46.3249 seconds and the one for sliding mode control is 73.2095 seconds. In the Case D, as shown in Fig. 9, the settling time for disturbance accommodating control is 46.2921 seconds and the one for sliding mode control is 72.8977 seconds. These simulation results clearly show that combining sliding mode control with a

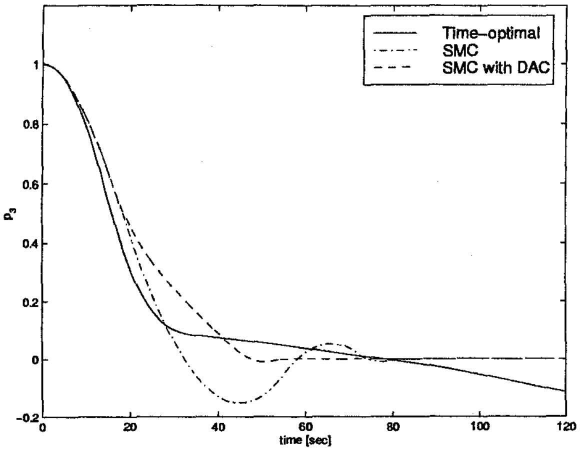

Fig. 9 Case D: Trajectories of the p_3

disturbance accommodating observer provides robustness with respect to inertia uncertainties and external distrubances.

CONCLUSION

Sliding mode control with PWPF was implemented and compared with the time-optimal solution. The settling times were minimized by tuning the gains in the sliding mode control. They were larger than 40% with respect to the ones of time-optimal solution. We also derived the upper bound of the sliding function for inertia uncertainty and external disturbances. A method for compensating the steady-state error of sliding mode control due to inertia uncertainty and external disturbance was presented and applied to spacecraft attitude maneuvers. The designed disturbance accommodating observer is linear allowing the use of many design and analysis methods for linear systems. When we use a continuous torque actuator, i.e., reaction wheels, the presented disturbance accommodating sliding mode control is more effective than the traditional sliding surface stabilizing problem since steady-state errors are reduced. Also, the robustness of sliding mode is guaranteed in the range of actuator capability. In this paper, the continuous torque actuator control was expanded to use PWPF thrusters. Simulation results show that the settling times were shorter than the ones of the sliding mode control with PWPF.

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