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FAST INTEGER AMBIGUITY RESOLUTION FOR GPS ATTITUDE DETERMINATION

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Abstract

In this paper, a new algorithm for GPS integer ambiguity resolution is shown. The algorithm first incorporates an instantaneous (static) integer search to significantly reduce the search space using a geometric inequality. Then a batch-type loss function is used to check the remaining integers in order to determine the optimal integer. This batch function represents the GPS sightline vectors in the body frame as the sum of two vectors, one depending on the phase measurements and the other on the unknown integers. The new algorithm has several advantages: it does not require an a-priori estimate of the vehicle's attitude; it provides an inherent integrity check using a covariance-type expression; and it can resolve the integers even when coplanar baselines exist. The performance of the new algorithm is tested on a dynamic hardware simulator.

Introduction

The use of phase difference measurements from Global Positioning System (GPS) receivers provides a novel approach for three-axis attitude determination and/or estimation. These measurements have been successfully used to determine the attitude of air-based,[1] space-based,[2-3] and sea-based[4] vehicles. Since phase differences are used, the correct number of integer wavelengths between a given pair of antennas must be found. The integer ambiguities can be determined using either "instantaneous" (motionless) or "dynamic" (motion-based) techniques. The ambiguities essentially act as integer biases to the phase difference measurements. Once the integer ambiguities are resolved, then the attitude determination problem can be solved.[5]

Instantaneous methods find a solution that minimizes the error residual at a specific time by searching through an exhaustive list of all possible integers and rejecting candidate solutions when the residual becomes too large.[6] Refinements can be made to the solution by restricting the search space with knowledge of a-priori information, such as the maximum tilt the baseline should encounter.[7] Instantaneous methods generally rely on solving a set of Diophantine equations.[8] The appeal of these methods is that they provide an "instantaneous" attitude solution, limited only by computation time, and are well suited to short baselines. However, the minimum residual does not guarantee a correct solution in the presence of noise.[9] In fact, it is possible that instantaneous methods can report a wrong solution as valid. This lack of integrity can cause significant problems if the sensor output is used to control a high bandwidth actuator, such as gas jets on a spacecraft. Another consideration is that instantaneous methods sometime require that the antenna array must be within a defined angle (typically 30 degrees) of a reference attitude, which is often true for ground-based applications, but is less likely for space-based applications. All of the aforementioned limitations imply that instantaneous methods, while attractive because of their fast solutions, are not totally acceptable for general purpose applications.

Dynamic techniques for resolving integer ambiguities involve collecting data for a given period of time and performing a batch solution, in which the integer terms remain constant over the collection period. These techniques rely on the fact that a certain amount of motion has occurred during the data collection, either from vehicle body rotation or GPS line of sight motion. Their main disadvantage, compared to instantaneous approaches, is that it takes time for the motion to occur, which may be on the order of several minutes. Another consideration is that these techniques may require large matrix inversions, which can cause numerical errors. But, motion-based techniques also have significant advantages over instantaneous methods. Most importantly, motion-based techniques are inherently high integrity methods because there are numerous checks that can be implemented into the solution before it is accepted. These include using statistical checks applied to error residuals, matrix condition number checks, and using the closeness of the computed floating-point "integers" to actual integers as a check. The probability of an erroneous solution being reported as valid can be made as small as desired by appropriately setting the thresholds on these integrity checks. For these reasons, motion-based techniques are considered to be more robust for on-board applications.

Traditional motion-based techniques of integer ambiguity resolution rely on the fact that either GPS line of sight motion or vehicle motion dominates the changes in differential carrier phase measurements. Cohen[9] developed an algorithm, known as "quasi-static" integer resolution, that can be used when the GPS line of sight motion and the vehicle rotation both account approximately evenly for the differential carrier phase measurement changes. This algorithm can be adapted to almost any vehicle motion, slow or fast, simply by varying the sample rate and the data collection time. The quasi-static method solves a collection of differential phase measurements for a single attitude estimate and then considers perturbations to the initial estimate at each measurement epoch to produce a time varying batch solution to the data. Although this is a widely used algorithm, there are certain disadvantages. First, an a-priori attitude estimate must be given. Second, the algorithm is an iterative batch estimator that may produce erroneous estimates, depending on the accuracy of the a-priori attitude estimate. Finally, if a large number of samples in the data collection are required to observe the motion, large-order matrix inversions may be required. Another method (Ref. 10) performs a minimization on three Euler-angle attitude parameters independent of each other, followed by determining the integers. This approach has been shown to provide better convergence than Cohen's method and works well for non-coplanar baselines; however, singular conditions can exist at various attitude rotations and a significant amount of vehicle motion may be necessary for a solution.

A new motion-based algorithm has been recently derived (Ref. 11), which has been shown to have significant advantages over prior methods, including: (i) it resolves the integer ambiguities without any a-priori attitude knowledge, (ii) it requires less computational effort, since large matrix inverses are not needed, and (iii) it is non-iterative. A disadvantage of the new algorithm is that it requires at least three non-coplanar baselines. The algorithm was first shown as a batch solution, and then shown as a sequential solution. A covariance expression has also been derived which can be used to bound the integer solution so that a sufficient integrity check for convergence can be developed. This is extremely useful in the sequential formulation, since the solution can be found as the motion occurs, rather than taking a batch solution at a specific data collection interval. However, a significant amount of vehicle motion is still required in order for the integers to be observable. In this paper, the aforementioned approach is expanded upon to use integer searches. Also, the case of three coplaner baselines is addressed.

This paper is organized as follows. First, the concept of the GPS phase difference measurement is introduced. Next, a geometric inequality is introduced that will be used to significantly reduce the integer search space. Then, the batch-type loss function used to resolve the remaining integers is shown, along with a covariance integrity check. Finally, the new algorithm is validated by using an actual GPS receiver with a hybrid dynamic simulator to simulate the vehicle motions of a low-altitude Earth-orbiting spacecraft.

GPS Sensor Model

In this section, a brief background of the GPS phase difference measurement is shown. The main measurement used for attitude determination is the phase difference of the GPS signal received from two antennas separated by a baseline. The wavefront angle and wavelength are used to develop a phase difference, as shown in Figure 1. The phase difference measurement is obtained by[9]

$$b_l \cos\theta = \lambda\left(\Delta\phi - n\right) \tag{1}$$

where b_l is the baseline length (in cm), θ is the angle between the baseline and the line of sight to the GPS spacecraft, n is the integer part of the phase difference between two antennae, $\Delta\phi$ is the fractional phase difference (in cycles), and λ is the wavelength (in cm) of the GPS signal. The two GPS frequency carriers are L1 at 1575.42 MHz and L2 at 1227.6 MHz. As of this writing, non-military applications generally use the L1 frequency. The measured fractional phase difference can be expressed by

$$\Delta\phi = \underline{b}^T A \underline{s} + n \tag{2}$$

where $\underline{s} \in R^3$ is the normalized line of sight vector to the GPS spacecraft in an external reference frame, $\underline{b} \in R^3$ is the baseline vector (in wavelengths), which is the relative position vector from one antenna to another expressed in the vehicle body frame, and $A \in R^{3\times 3}$ is the attitude matrix, an orthogonal matrix with determinant 1 (i.e., $A^T A = I_{3\times 3}$) representing the transformation between the two frames. The measurement model is given by

$$\Delta\tilde{\phi}_{ij} = \underline{b}_i^T A \underline{s}_j + n_{ij} + w_{ij} \tag{3}$$

where $\Delta\tilde{\phi}_{ij}$ denotes the phase difference measurement for the i^{th} baseline and j^{th} sightline, and w_{ij} represents a zero-mean Gaussian measurement error with standard deviation ϖ_{ij} which is $0.5\,\text{cm}/\lambda = 0.026$ wavelengths for typical phase noise.[9]

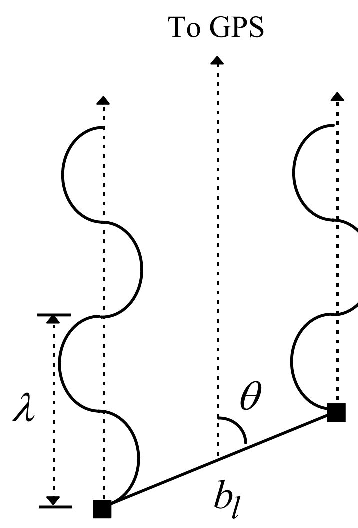

Fig. 1 GPS Wavelength and Wavefront Angle

Integer Ambiguity Resolution

In this section a new attitude-independent algorithm to resolve the integer ambiguities is presented using static searches. This involves using a series of tests that the possible integers must first pass, which is used to significantly reduce the search space. Then, an optimal batch-type loss function is minimized to determine the optimal integers.

Static algorithms have an advantage in that they provide an instantaneous solution of the integers. However, they are prone to noise errors, which can induce incorrect solutions. In this paper an integer search is performed to maximize the probability that a unique solution is the correct solution, while at the same time reducing the search space by using normality constraints as well as geometric constraints. First, it assumed that either three noncoplanar baselines or three noncoplanar sightlines are available (if three noncoplanar baselines exist then they should be used). The first step involves reducing the integer search space by using a subset of only two baselines and two sightlines. With this subset, a significant reduction in the search space is possible (especially for long baselines). For example, with three baselines (assuming κ possible integers associated with each baseline) the search space required to determine the integers is on the order of κ^3; however, with the reduced subset the search space is now on the order of $3\kappa^2$. For this reduced subset, it can be shown from geometry that the following inequality must be true (using baselines $\underline{b}_1$ and $\underline{b}_2$)

$$\begin{aligned}\|\underline{b}_1\|^2 \|\underline{b}_2\|^2 > (\underline{b}_1 \cdot \underline{b}_2)^2 + \|\underline{b}_2\|^2 \left(\Delta\tilde{\phi}_{1j} - n_{1j}\right)^2 \\ -2\left(\Delta\tilde{\phi}_{1j} - n_{1j}\right)\left(\Delta\tilde{\phi}_{2j} - n_{2j}\right)(\underline{b}_1 \cdot \underline{b}_2) \\ + \|\underline{b}_1\|^2 \left(\Delta\tilde{\phi}_{2j} - n_{2j}\right)^2\end{aligned} \tag{4}$$

Note, the same inequality can be applied using sightlines $\underline{s}_1$ and $\underline{s}_2$:

$$\begin{aligned}\|\underline{b}_i\|^2 > \|\underline{b}_i\|^2 (\underline{s}_1 \cdot \underline{s}_2)^2 + \left(\Delta\tilde{\phi}_{i1} - n_{i1}\right)^2 \\ -2\left(\Delta\tilde{\phi}_{i1} - n_{i1}\right)\left(\Delta\tilde{\phi}_{i2} - n_{i2}\right)(\underline{s}_1 \cdot \underline{s}_2) \\ + \left(\Delta\tilde{\phi}_{i2} - n_{i2}\right)^2\end{aligned} \tag{5}$$

If the integers have been properly resolved then it can be shown that Equation (4) reduces down to (in the noise free case)

$$\left[(A\underline{s}_j) \cdot (\underline{b}_1 \times \underline{b}_2)\right]^2 > 0 \tag{6}$$

This means that $A\underline{s}_j$, $\underline{b}_1$ and $\underline{b}_2$ must not lie in the same plane. We need this condition to be able to extract attitude information outside of the $\underline{b}_1, \underline{b}_2$ plane. The triple scalar product, $(A\underline{s}_j) \cdot (\underline{b}_1 \times \underline{b}_2)$, is the signed volume of the parallelepiped spanned by the vectors $A\underline{s}_j$, $\underline{b}_1$, $\underline{b}_2$; where the sign is positive, if and only if, the vectors form a right-handed system. This is shown geometrically in Figure 2. The height of the parallelepiped is given $\left\|A\underline{s}_j\right\|\cos\angle\left(\underline{b}_1 \times \underline{b}_2, A\underline{s}_j\right)$, and the base area is given by $\left\|\underline{b}_1 \times \underline{b}_2\right\|$. Note, Equation (6) is almost always satisfied if the integers pass the test in Equation (4). Equation (4) or (5) can be used to significantly reduce the search space, since only two baselines (or two sightlines) are considered at a time, as opposed to considering all three simultaneously.

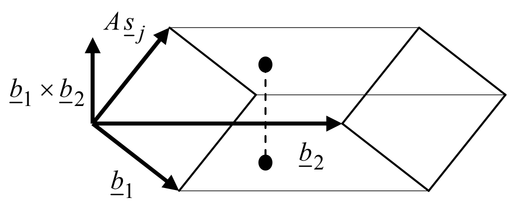

Fig. 2 Parallelepiped Formed by Three Vectors

The next step involves converting the sightlines into the body frame or converting the baselines into the reference frame. For the former the algorithm begins by representing the *j*th sightline vector in the body frame, $A\underline{s}_j$, as the sum of two components. The first component $\hat{\underline{s}}_j$ is a function of the measured fractional phase measurements, and the second $\underline{c}_j$ depends on the unknown integer phase differences. This representation is accomplished by minimizing the following loss function

$$J\left(A\underline{s}_j\right) = \frac{1}{2}\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\left(\Delta\tilde{\phi}_{ij} - n_{ij} - \underline{b}_i^T A\underline{s}_j\right)^2 \quad \text{for } j = 1, 2, \ldots, N \tag{7}$$

where M is the number of baselines and N is the number of available sightlines. If at least three non-coplanar baselines exist, the minimization of Equation (7) is straightforward and leads to

$$A\underline{s}_j = \hat{\underline{s}}_j - \underline{c}_j \tag{8a}$$

$$\hat{\underline{s}}_j = B_j^{-1}\left[\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\Delta\tilde{\phi}_{ij}\,\underline{b}_i\right] \tag{8b}$$

$$\underline{c}_j = B_j^{-1}\left[\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}n_{ij}\,\underline{b}_i\right] \tag{8c}$$

$$B_j = \sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\underline{b}_i\underline{b}_i^T \tag{8d}$$

Since the measurements are not perfect, Equation (8a) is replaced by the effective measurement model

$$\hat{\underline{s}}_j = A\underline{s}_j + \underline{c}_j + \underline{\varepsilon}_j \tag{9}$$

where $\underline{c}_j$ is a constant bias since the baselines are assumed constant, and $\underline{\varepsilon}_j$ is a zero-mean Gaussian process with covariance $R_j = B_j^{-1}$. This model is used for the actual attitude determination,[12] which we will not consider further in this paper.

The next step is to use an attitude-independent method to find the phase-bias vector $\underline{c}_j$ for each sightline, which gives all the sightlines in both the body frame and the reference frame. The explicit integer phases are not needed for this solution, but it is important to check that they are close to integer values, as mentioned in the Introduction. In the general case, the explicit integer phases can be found from the attitude solution. The three-baseline case ($M = 3$) is simpler, for in this case Equation (8c) can be inverted to give

$$n_{ij} = \underline{b}_i^T\,\underline{c}_j \tag{10}$$

With more than three baselines, however, Equation (8c) does not have a unique solution for $\underline{c}_j$, so the M integer phases for sightline $\underline{s}_j$ cannot be found from $\underline{c}_j$ alone. We will consider the three-baseline case, which is the most common in practice. If more baselines are available, we are always free to select a three-baseline subset. Then, after the integer phases have been determined, a refined attitude estimate can be computed using all baselines (i.e., three baselines are sufficient to determine an attitude, which may then be used to resolve the integers corresponding to the other baselines).

To eliminate the dependence on the attitude, the orthogonality of A and Equation (9) are used to give

$$\begin{aligned}\left\|\underline{s}_j\right\|^2 = \left\|A\underline{s}_j\right\|^2 &= \left\|\hat{\underline{s}}_j - \underline{c}_j - \underline{\varepsilon}_j\right\|^2 \\ &= \left\|\hat{\underline{s}}_j\right\|^2 - 2\hat{\underline{s}}_j\cdot\underline{c}_j + \left\|\underline{c}_j\right\|^2 \\ &\quad -2\left(\hat{\underline{s}}_j - \underline{c}_j\right)\cdot\underline{\varepsilon}_j + \left\|\underline{\varepsilon}_j\right\|^2\end{aligned} \tag{11}$$

Next, following Alonso and Shuster, the following effective measurement and noise are defined[13]

$$z_j \equiv \left\|\hat{\underline{s}}_j\right\|^2 - \left\|\underline{s}_j\right\|^2 \tag{12a}$$

$$v_j \equiv 2\left(\hat{\underline{s}}_j - \underline{c}_j\right)\cdot\underline{\varepsilon}_j - \left\|\underline{\varepsilon}_j\right\|^2 \tag{12b}$$

Then, the effective measurement model is

$$z_j = 2\underline{\hat{s}}_j \cdot \underline{c}_j - \left\|\underline{c}_j\right\|^2 + v_j \tag{13}$$

where v_j is approximately Gaussian for small $\underline{\varepsilon}_j$ with mean and variance given by

$$\mu_j \equiv E\left\{v_j\right\} = -\text{trace}\left\{R_j\right\} \tag{14}$$

and

$$\sigma_j^2 \equiv E\left\{v_j^2\right\} - \mu_j^2 = 4\left(\underline{\hat{s}}_j - \underline{c}_j\right)^T R_j \left(\underline{\hat{s}}_j - \underline{c}_j\right) - \mu_j^2 \tag{15}$$

respectively. Equations (12)-(15) define an attitude-independent algorithm because they do not contain the attitude matrix *A*.

The negative-log-likelihood function for the bias is given by

$$J\left(\underline{c}_j\right) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\sigma_j^2(k)}\left[z_j(k) - 2\underline{\hat{s}}_j(k)\cdot\underline{c}_j + \left\|\underline{c}_j\right\|^2 - \mu_j(k)\right]^2 + \log\sigma_j^2(k) + \log 2\pi\right\} \tag{16}$$

where L is the total number of measurement epochs, and the symbol k denotes the variable at time t_k. The maximum-likelihood estimate for $\underline{c}_j$, denoted by $\underline{c}_j^*$, minimizes the negative-log-likelihood function, and satisfies

$$\left.\frac{\partial J\left(\underline{c}_j\right)}{\partial \underline{c}_j}\right|_{\underline{c}_j^*} = \underline{0} \tag{17}$$

The minimization of Equation (16) is not straightforward since the likelihood function is quartic in $\underline{c}_j$. A number of algorithms have been proposed for estimating the bias (see Ref. 12 for a survey). The simplest solution is obtained by scoring, which involves a Newton-Raphson iterative approach. Another approach avoids the minimization of a quartic loss function by using a "centered" estimate. A statistically correct centered estimate is also derived in Ref. 13. Furthermore, Alonso and Shuster show a complete solution of the statistically correct centered estimate that determines the exact maximum likelihood estimate $\underline{c}_j^*$. This involves using the statistically correct centered estimate as an initial estimate, and iterating on a correction term using a Gauss-Newton method. Although this extension to the statistically correct centered estimate can provide some improvements, this part is not deemed necessary for the GPS problem since the estimated quantity for n_{ij} is rounded to the nearest integer.

In this paper another approach is used. As before, we consider the case for $M = 3$. Equations (8b) and (8c) are first re-written as

$$\underline{\hat{s}}_j = B_j^{-1}\,\Gamma_j\,\underline{\Phi}_j \tag{18a}$$

$$\underline{c}_j = B_j^{-1}\,\Gamma_j\,\underline{n}_j \tag{18b}$$

where

$$\Gamma_j \equiv \begin{bmatrix} \varpi_{1j}^{-2}\,\underline{b}_1 & \varpi_{1j}^{-2}\,\underline{b}_2 & \varpi_{1j}^{-2}\,\underline{b}_3 \end{bmatrix} \tag{19a}$$

$$\underline{n}_j \equiv \begin{bmatrix} n_{1j} \\ n_{2j} \\ n_{3j} \end{bmatrix} \tag{19b}$$

$$\underline{\Phi}_j \equiv \begin{bmatrix} \Delta\tilde{\phi}_{1j} \\ \Delta\tilde{\phi}_{2j} \\ \Delta\tilde{\phi}_{3j} \end{bmatrix} \tag{19c}$$

The loss function in Equation (16) can now be re-written (neglecting the term independent of $\underline{n}_j$):

$$J\left(\underline{n}_j\right) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\sigma_j^2(k)}\left[\left\|B_j^{-1}\Gamma_j\left(\underline{\Phi}_j(k) - \underline{n}_j\right)\right\|^2 - \left\|\underline{s}_j(k)\right\|^2 + \text{trace}\left\{B_j^{-1}\right\}\right]^2 + \log\sigma_j^2(k)\right\} \tag{20}$$

with

$$\sigma_j^2(k) = \left(\underline{\Phi}_j(k) - \underline{n}_j\right)^T \Gamma_j^T B_j^{-3}\,\Gamma_j\left(\underline{\Phi}_j(k) - \underline{n}_j\right) - \text{trace}^2\left\{B_j^{-1}\right\} \tag{21}$$

Equation (20) can now be used to directly determine the integers without pre-computing the sightline vector in the body frame. Equation (20) clearly indicates that the loss function involves a scalar check on the norm vector residuals (since $B_j^{-1}\Gamma_j\left(\underline{\Phi}_j - \underline{n}_j\right) = \underline{\hat{s}}_j - \underline{c}_j$).[14] In practice if $\underline{n}_j$ is real valued, then a sufficient amount of vehicle motion must occur in order to determine the minimum. This was the approach used in Ref. [11]. However, the solution in this paper involves checking the remaining integers that have passed the inequality condition in Equation (4). Since the solutions for the components of $\underline{n}_j$ are constrained to be integers, then it is more likely that a unique solution which minimizes Equation (21) can be determined with minimal vehicle motion.

The estimate error covariance can also be computed in order to insure that the determined integer is statistically correct. This can be shown to be given in the limit of infinitely large samples by

$$P_j = \left\{ \sum_{k=1}^{L} \frac{4}{\sigma_j^2(k)} \left[\underline{\Phi}_j(k) - \underline{n}_j \right] \left[\underline{\Phi}_j(k) - \underline{n}_j \right]^T \right\}^{-1} \tag{22}$$

Equation (22) can be used to develop an integrity check for the algorithm, using standard results on hypothesis testing.[15] For example, the computed integer can be shown to have only a 0.0013 probability of selecting the wrong integer when three times the square root of a diagonal element of P_j is less than 1/2.

The case where 3 coplanar baselines can be determined by considering 3 non-coplanar sightlines. A batch solution for this case can be determined using a similar approach shown in Ref. 11; however, the statistically correct centered estimate approach is complex since the sightlines vary with time. Also, this requires that the same 3 sightlines are available long enough to determine a solution (which isn't always possible). The integer search approach presented here may alleviate these difficulties. The loss function for this case is given by

$$\begin{aligned} J(\underline{n}_i) = \frac{1}{2} \sum_{k=1}^{L} \Bigg\{ \frac{1}{\sigma_i^2(k)} \Bigg[\left\| S_i^{-1}(k) \Gamma_i(k) \left(\underline{\Phi}_i(k) - \underline{n}_i \right) \right\|^2 \\ - \left\| \underline{b}_i \right\|^2 + \text{trace}\left\{ S_i^{-1}(k) \right\} \Bigg]^2 + \log \sigma_i^2(k) \Bigg\} \end{aligned} \tag{23}$$

where

$$\begin{aligned} \sigma_i^2(k) = -\text{trace}^2 \left\{ S_i^{-1}(k) \right\} \\ + \left(\underline{\Phi}_i(k) - \underline{n}_i \right)^T \Gamma_i^T(k) S_i^{-3}(k) \Gamma_i(k) \left(\underline{\Phi}_i(k) - \underline{n}_i \right) \end{aligned} \tag{24a}$$

$$\Gamma_i \equiv \left[\varpi_{i1}^{-2}\, \underline{s}_1 \quad \varpi_{i2}^{-2}\, \underline{s}_2 \quad \varpi_{i3}^{-2}\, \underline{s}_3 \right] \tag{24b}$$

$$\underline{n}_i \equiv \begin{bmatrix} n_{i1} \\ n_{i2} \\ n_{i3} \end{bmatrix} \tag{24c}$$

$$\underline{\Phi}_i \equiv \begin{bmatrix} \Delta\tilde{\phi}_{i1} \\ \Delta\tilde{\phi}_{i2} \\ \Delta\tilde{\phi}_{i3} \end{bmatrix} \tag{24d}$$

$$S_i = \varpi_{i1}^{-2}\, \underline{s}_1 \underline{s}_1^T + \varpi_{i2}^{-2}\, \underline{s}_2 \underline{s}_2^T + \varpi_{i3}^{-2}\, \underline{s}_3 \underline{s}_3^T \tag{24e}$$

For this case both Γ and S vary with time. The estimate error covariance for this case is given by

$$P_i = \left\{ \sum_{k=1}^{L} \frac{4}{\sigma_i^2(k)} \left[\underline{\Phi}_i(k) - \underline{n}_i \right] \left[\underline{\Phi}_i(k) - \underline{n}_i \right]^T \right\}^{-1} \tag{25}$$

Once again, this can be used to develop an integrity check for the algorithm.

There are many advantages of the new algorithm. First, the algorithm is fully autonomous (i.e., it requires no a-priori information such as an a-priori attitude guess). Second, it can be used to determine the integers when 3 coplanar baselines exist. Third, the required search space can be significantly reduced using Equation (4) or (5). Finally, the integers for other sightlines (or baselines) can be easily resolved by calling the same subroutine. For these reasons, the new algorithm provides an attractive approach to resolve the integers.

Hardware Simulation and Results

A hardware simulation of a typical spacecraft attitude determination application was undertaken to demonstrate the performance of the new algorithm. For this simulation, a Northern Telecom 40 channel, 4 RF output STR 2760 unit was used to generate the GPS signals that would be received at a user specified location and velocity. The signals are then provided directly (i.e., they are not actually radiated) to a GPS receiver that has been equipped with software tracking algorithms that allow it operate in space (see Figure 3 for details).

The receiver that was used was a Trimble TANS Vector; which is a 6 channel, 4 RF input multiplexing receiver that performs 3-axis attitude determination using GPS carrier phase and line of sight measurements. This receiver software was modified at Stanford University and NASA-Goddard to allow it to operate in space. This receiver model has been flown and operated successfully on several spacecraft, including: REX-II, OAST-Flyer, GANE, Orbcomm, Microlab, and others.

The simulated motion profile was for an actual spacecraft, the Small Satellite Technology Initiative (SSTI) Lewis satellite, which carried an experiment to assess the performance of GPS attitude determination on-orbit. Although the spacecraft was lost due to a malfunction not related to the GPS experiment shortly after launch, this motion profile is nonetheless very representative of the types of attitude determination applications. The orbit parameters and pointing profile used for the simulation are given in Table 1.

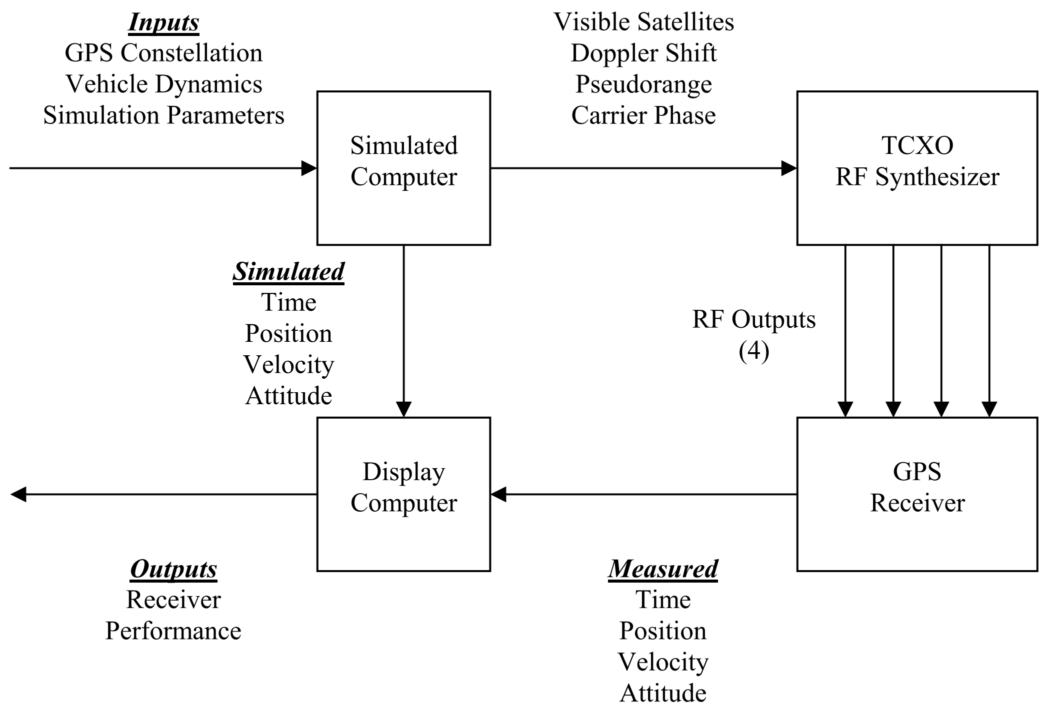

Fig. 3 Hardware Simulation Block Diagram

Table 1 SSTI Lewis Orbit parameters

Semimajor axis (a)	6901.137 km
Inclination (i)	97.45 deg
Right Ascension of Ascending Node	-157.1 deg
Eccentricity (e)	0.0001
Pointing profile	Earth pointed
Launch date	Aug. 22, 1997

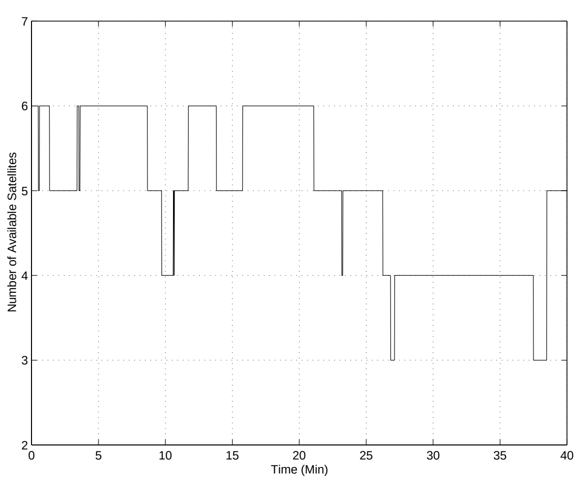

Fig. 4 Number of Available GPS Spacecraft

The simulated SSTI Lewis spacecraft has four GPS antennas that form three baselines. The antenna separation distances are 0.61 m, 1.12 m, and 1.07 m, respectively. One antenna (in baseline 3) is located 0.23 m out of plane (below) the other three antennas. On the spacecraft, the antennas are mounted on pedestals with ground planes to minimize signal reflections and multipath. For the simulation, multipath errors are introduced using a simple Markov-process with time constant of 5 minutes and standard deviation of 0.026 wavelengths.[16] The baseline vectors in wavelengths are given by

$$\underline{b}_1 = \begin{bmatrix} 2.75 \\ 1.64 \\ -0.12 \end{bmatrix}, \quad \underline{b}_2 = \begin{bmatrix} 0.00 \\ 6.28 \\ -0.17 \end{bmatrix}, \quad \underline{b}_3 = \begin{bmatrix} -3.93 \\ 3.93 \\ -1.23 \end{bmatrix} \tag{26}$$

Line biases are first determined before the new algorithm is tested to resolve the integer ambiguities. The GPS raw measurements are processed at 1 Hz over a forty minute simulation. A plot of the number of available GPS spacecraft for the simulated run is shown in Figure 4. During the simulated run, a minimum of three visible GPS satellites are in sight at all times. However, resolution of the ambiguous integers for the phase measurements from any specific GPS satellite requires that it remain in view continuously until the sequential algorithm converges. In practice, all available sightlines should be processed, since attitude determination requires the integers to be resolved for two GPS satellites simultaneously. The simulation contains a number of eight minute spans when sightlines to two specific GPS satellites are continuously available for the ambiguity resolution algorithm.

Since the baselines are non-coplanar, Equations (4) and (20) will used to determine the integers. For the simulation the following integer ambiguities are introduced:

$$\underline{n}_1 = \begin{bmatrix} -6 \\ 1 \\ 3 \end{bmatrix}, \quad \underline{n}_2 = \begin{bmatrix} 5 \\ -8 \\ -2 \end{bmatrix} \tag{27}$$

From the baseline geometry the integers are bounded from –8 to 8. If a full search is implemented using Equation (20) solely this would require 8192 searches (4096 for each sightline). Equation (4) requires a total of only 1024 searches. The integers at the initial time that passed the inequality in Equation (4) are shown in Table 2.

Table 2 Remaining Integers and Associated Costs

	Integers	Cost
First Baseline	-6 1 3	66.2
	-3 5 3	125.6
	-3 7 3	146.3
	-1 8 2	1862.3
Second Baseline	5 -8 -2	64.0
	7 -6 -3	119.5
	7 -5 -3	136.5
	8 -8 -8	209.6
	8 -7 -7	547.4

Table 2 clearly shows how Equation (4) can be used to dramatically reduce the search space. Only 9 remaining searches need to be used in Equation (20). Since the computational load to do this search is extremely low, a search can be implemented in a very efficient manner, periodically in the background. For this simulation the cost function in Equation (20) is summed over time and has been checked every 30 seconds. The stopping condition is given when three times the square root of every diagonal element of P_j is less than 1/2.[11] For this case, this occurred in 2 minutes for each sightline. The associated cost for each integer set after 2 minutes is also shown in Table 2. Clearly, the correct integers have been found. This has clear advantages over the motion-based technique in Ref. [11], which required over 6 minutes for the solution to converge. Also, another simulation using Equations (5) and (23) has been performed in order to investigate the performance using 3 non-coplanar sightlines. Results indicate that this approach give comparable results to the results shown in Table 2.

Conclusions

In this paper, a new algorithm was developed for GPS integer ambiguity resolution. The algorithm uses the best qualities of both instantaneous and motion-based techniques. It uses an instantaneous approach to substantially reduce the search space, and then uses a batch-type loss function to resolve the remaining possible integers. The new algorithm has several advantages over previously existing algorithms. First, the algorithm is attitude independent so that no a-priori attitude estimate (or assumed vehicle motion) is required. Second, a suitable integrity check can be used to determine the correct values. Finally, it can resolve the integers even when coplanar baselines exist. The algorithm was tested using a GPS hardware simulator to simulate the motions of a typical low-altitude Earth-orbiting spacecraft. Results indicated that the new algorithm provides a viable and attractive means to effectively resolve the integer ambiguities.

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