# Autonomous Attitude Determination for ISS Applications Using Pseudolite Signals

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## Abstract

In this paper, the utilization of Global Positioning System type transmitters, called pseudolites, for fully autonomous vehicle-to-vehicle relative attitude determination is explored. Several issues are investigated in order to provide real-time, autonomous, and reliable attitude information using pseudolites. These issues include: the effect of non-planar wavefronts due to making measurements from the near-field of pseudolites, and the resolution of the integer ambiguities which arise since phase difference measurements are used to determine the attitude. First, results of a simulation study show how the non-planar wavefronts affect the attitude accuracy. Then, a new algorithm for GPS integer ambiguity resolution is shown. The new algorithm has several advantages: it does not require an a-priori estimate of the vehicle's attitude; it provides an inherent integrity check using a covariance-type expression; and it can resolve the integers even when coplanar baselines exist. Using an autonomous relative attitude system has many advantages for applications on the International Space Station, including: the reduction of the likelihood of re-contact in the Crew Return Vehicle separation scenario, and the use in providing relative attitude for rendezvous and docking between other space vehicles.

## Introduction

Real-time knowledge of the relative attitude between two vehicles is, in general, a requirement in situations in which the vehicles must be aligned properly to carry out some objective. For instance, two space vehicles performing an on-orbit rendezvous, approach, and docking would particularly benefit from attitude knowledge for autonomous docking guidance. Moreover, in a "blind separation" situation, such as the scenario currently being considered for the Crew Return Vehicle (CRV) as it undergoes an emergency departure from the International Space Station (ISS), relative attitude knowledge would greatly enhance the likelihood of autonomous separation without re-contact (i.e., collision) between the two vehicles.

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The latter scenario of an emergency departure of the CRV from the ISS has an added component of difficulty in that current plans for the CRV do not include relative measurements of any sort between the -- presumably completely disabled -- ISS. Thus, current plans call for the CRV to use instruments that provide only absolute attitude knowledge in an Earth-centered frame. Absolute attitude will be measured on the CRV by a 3-axis ring laser gyro unit, an attitude-determination-capable GPS receiver, and possibly a magnetometer. The only knowledge of CRV-to-ISS relative attitude (e.g., in the ISS structural frame) will be the initial alignment of the CRV on its ISS docking ring. In the emergency departure, particularly during the crucial first few seconds before the first propulsive separation burn, the CRV attitude will be based on gyro rate-based dead reckoning, with no measurements of the changing ISS attitude.

In this paper, the use of an autonomous (independent of ISS power or data systems) set of radio transmitters is explored, to enable rapid and precise (sub-1°) vehicle-to-vehicle relative attitude determination. The attitude determination would be based on near-field radio signals from these transmitters, called pseudolites,[1] which are radio-frequency transmitters that emit signals similar to the Global Positioning System (GPS) transmissions.[2-4] While pseudolites are still a developing technology, which have not yet been demonstrated in an on-orbit space flight experiment, it is perceived that they hold great promise for augmenting the existing GPS satellite-based navigation and attitude determination technology in local-area applications. At the NASA Johnson Space Center (JSC), a recent project demonstrated the use of an indoor pseudolite "constellation" to enable autonomous position determination in a robot. There are plans to continue development and research related to pseudolite-based navigation at the new JSC Navigation Systems & Technology Laboratory (NSTL).

The problem of finding the attitude of a vehicle using GPS-type signals essentially involves a two-step process. First, since phase differences are used, the correct number of integer wavelengths between a given pair of antennas must be found. This problem can generally be solved using instantaneous integer searches or using motion-based techniques. Much attention has been placed on resolving the integer ambiguity problem over the past many years. Once the integer ambiguities are known, then the attitude problem must be solved.[5]

Instantaneous methods[6,7] for integer ambiguity resolution find a solution that minimizes the error residual at a specific time by searching through an exhaustive list of all possible integers and rejecting candidate solutions when the residual becomes too large. Refinements can be made to the solution by restricting the search space with knowledge of a-priori information, such as the maximum tilt the baseline should encounter. Instantaneous methods generally rely on solving a set of Diophantine equations.[8] The appeal of these methods is that they provide an "immediate" attitude solution, limited only by computation time, and are well suited to short baselines. However, the minimum residual does not guarantee a correct solution in the presence of noise. In fact, it is possible that instantaneous methods can report a wrong solution as valid. This lack of integrity can cause significant problems if the sensor output is used to control a high bandwidth actuator, such as gas jets on a spacecraft. Another consideration is that instantaneous methods sometime require that the antenna array must be within a defined angle (typically 30 degrees) of a reference attitude, which is often true for ground-based applications, but is less likely for space-based applications.

Dynamic techniques for resolving integer ambiguities involve collecting data for a given period of time and performing a batch solution,[9-11] in which the integer terms remain constant over the collection period. These techniques rely on the fact that a certain amount of motion has occurred during the data collection, either from vehicle body rotation or GPS line of sight motion. Their main disadvantage, compared to instantaneous approaches, is that it takes time for the motion to occur, which may be on the order of several minutes. Another consideration is that a potentially significant amount of memory is required for the storage of the batch data collection. But, motion-based techniques also have significant advantages over instantaneous methods. Most importantly, motion-based techniques are inherently high integrity methods because there are numerous checks that can be implemented into the solution before it is accepted. These include using statistical checks applied to error residuals, matrix condition number checks, and using the closeness of the computed floating-point "integers" to actual integers as a check. The probability of an erroneous solution being reported as valid can be made as small as desired by appropriately setting the thresholds on these integrity checks.

This paper is organized as follows. First, the concept of the GPS phase difference measurement is introduced. Then, the effect of non-planar wavefronts due to making measurements from the near-field of pseudolites is investigated. Next, the integer ambiguity problem is addressed. A geometric inequality is introduced that will be used to significantly reduce the integer search space. The batch-type loss function used to resolve the remaining integers is shown, along with a covariance integrity check. Finally, the new algorithm is tested using simulated measurements of vehicle motion relative to the ISS.

## GPS Sensor Model

In this section, a brief background of the GPS phase difference measurement is shown. The main measurement used for attitude determination is the phase difference of the GPS signal received from two antennas separated by a baseline. The wavefront angle and wavelength are used to develop a phase difference, as shown in Figure 1.

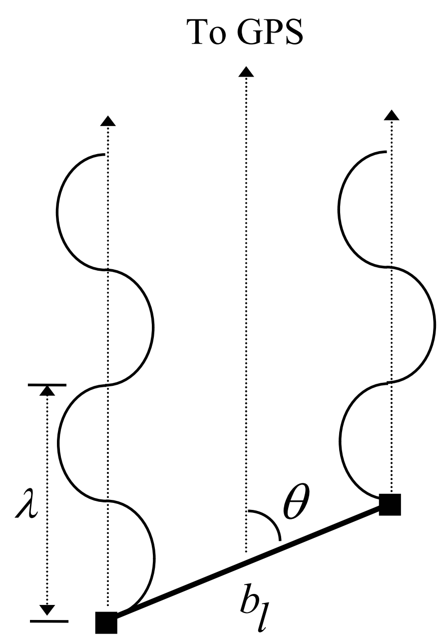


**Figure 1 GPS Wavelength and Wavefront Angle**

The phase difference measurement is obtained by[9]

$$b_l \cos\theta = \lambda(\Delta\phi - n) \quad (1)$$

where $b_l$ is the baseline length (in cm), $\theta$ is the angle between the baseline and the line of sight to the GPS spacecraft, $n$ is the integer part of the phase difference between two antennae, $\Delta\phi$ is the fractional phase difference (in cycles), and $\lambda$ is the wavelength (in cm) of the GPS signal. The two GPS frequency carriers are L1 at 1575.42 MHz and L2 at 1227.6 MHz. As of this writing, non-military applications generally use the L1 frequency. The measured fractional phase difference can be expressed by

$$\Delta\phi = \underline{b}^T A \underline{s} + n \quad (2)$$

where $\underline{s} \in R^3$ is the normalized line of sight vector to the GPS spacecraft in a reference frame, $\underline{b} \in R^3$ is the baseline vector (in wavelengths), which is the relative position vector from one antenna to another, and $A \in R^{3\times3}$ is the attitude matrix, an orthogonal matrix with determinant 1 (i.e., $A^T A = I_{3\times3}$) representing the transformation between the two frames. The measurement model is given by

$$\Delta\tilde{\phi}_{ij} = \underline{b}_i^T A \underline{s}_j + n_{ij} + w_{ij} \quad (3)$$

where $\Delta\tilde{\phi}_{ij}$ denotes the phase difference measurement for the $i^{\text{th}}$ baseline and $j^{\text{th}}$ sightline, and $w_{ij}$ represents a zero-mean Gaussian measurement error with standard deviation $\varpi_{ij}$ which is $0.5\,\text{cm}/\lambda = 0.026$ wavelengths for typical phase noise.[9]

## Attitude Determination in the Near Field of Pseudolites

The effect of non-planar wavefronts due to making measurements from the near-field of pseudolites is shown in an example. Consider a system containing two baselines, making observations of three non-coplanar sightlines. Moreover, for simplicity in this example, let us treat the phase observations as being unambiguous ranges, as if the integer ambiguities had already been determined.

A simulation has been performed to generate a set of single-difference phase measurements, based on transmitter distances ranging from 25 meters to 25 thousand kilometers. The two simulated baselines were both 3 meters long, and were orthogonal to each other. The transmitters were simulated in three cases. The first case modeled the transmitters at a GPS-like distance. The geometry of the three transmitters was such that they were widely distributed in the sky in this case. The second and third cases modeled the transmitters pseudolite-like distances, with spacing between the transmitters on the order of 2 to 20 meters. Table 1 shows the simulation parameter values.

The simulated single-differenced measurements were used to estimate the attitude of the "body" to which the baseline pair was attached. The estimation technique applied for all cases was an iterated minimum-variance algorithm, in which the measurements were modeled with the linearized expression for the unambiguous phase difference in Equation (3).

Table 2 shows the results from the simulation run. As expected, the attitude solution accuracy is essentially perfect (as we are using perfect measurements in this simulation) for cases in which the transmitters are far away. It is seen using the signal set from 250 meters distance results in accuracy on the order of 1 degree per axis. The signal set from 25 meters distance, results in accuracy on the order of 5 to 10 degrees per axis, based on this approach, due to the nonlinearity of the actual measurements.

**Table 1 Parameters for Simulation of Far-Field and Near-Field Signals**

| | |
|---|---|
| Case 1: Distant Transmitters<br>GPS-Like Scenario | Distance: $2.5 \times 10^7$ m<br>$(\text{Elev, Az})_{\text{Transmitter 1}} = (10°, 0°)$<br>$(\text{Elev, Az})_{\text{Transmitter 2}} = (10°, 120°)$<br>$(\text{Elev, Az})_{\text{Transmitter 3}} = (70°, 240°)$ |
| Case 2: Mid-Field Transmitters:<br>Pseudolite-Like Scenario, with 20-m-level distances between transmitters | Distance: $2.5 \times 10^2$ m<br>$(\text{Elev, Az})_{\text{Transmitter 1}} = (10°, 0°)$<br>$(\text{Elev, Az})_{\text{Transmitter 2}} = (15°, 8°)$<br>$(\text{Elev, Az})_{\text{Transmitter 3}} = (20°, 15°)$ |
| Case 3: Near-Field Transmitters:<br>Pseudolite-Like Scenario, with 2-m-level distances between transmitters | Distance: $2.5 \times 10^1$ m<br>$(\text{Elev, Az})_{\text{Transmitter 1}} = (10°, 0°)$<br>$(\text{Elev, Az})_{\text{Transmitter 2}} = (15°, 8°)$<br>$(\text{Elev, Az})_{\text{Transmitter 3}} = (20°, 15°)$ |

**Table 2 Attitude Solutions Based on Simulated Far-Field and Near-Field Signals**

| | **Roll** | **Pitch** | **Yaw** |
|---|---|---|---|
| **True Attitude** | 10.0° | -73.0° | 20.0° |
| **Case 1 Attitude Estimate** | 10.000017° | -73.000003° | 20.000014° |
| **Case 2 Attitude Estimate** | 10.502° | -74.087° | 20.118° |
| **Case 3 Attitude Estimate** | 16.428° | -78.558° | 23.051° |

The results of this simulation provide an interesting study of the degradation of attitude accuracy for a simple estimator, in the presence of non-planar wavefronts. Case 2 and particularly Case 3 show that while the nonlinearity of the single-differenced phase measurement has a deleterious effect in the pseudolite near field, the potential exists for attitude determination with few-degree accuracy per axis. However, this simulation has assumed that the phase ambiguities are already resolved. This motivates further investigation of ambiguity resolution methods for near-field phase-based attitude determination.

## Integer Ambiguity Resolution

In this section a new attitude-independent algorithm to resolve the integer ambiguities is presented using static searches. This involves using a series of tests that the possible integers must first pass, which is used to significantly reduce the search space. Then, an optimal batch-type loss function is minimized to determine the optimal integers.

Static algorithms have an advantage in that they provide an instantaneous solution of the integers. However, they are prone to noise errors, which can induce incorrect solutions. In this paper an integer search is performed to maximize the probability that a unique solution is the correct solution, while at the same time reducing the search space by using normality constraints as well as geometric constraints. First, it assumed that either three noncoplanar baselines or three noncoplanar sightlines are available (if three noncoplanar baselines exist then they should be used). The first step involves reducing the integer search space by using a set of only two baselines and two sightlines. For this case, it can be shown from geometry that the following inequality must be true (using baselines $\underline{b}_1$ and $\underline{b}_2$)

$$\begin{aligned}\|\underline{b}_1\|^2\|\underline{b}_2\|^2 > \left(\underline{b}_1\cdot\underline{b}_2\right)^2 + \|\underline{b}_2\|^2\left(\Delta\tilde{\phi}_{1j}-n_{1j}\right)^2 \\ -2\left(\Delta\tilde{\phi}_{1j}-n_{1j}\right)\left(\Delta\tilde{\phi}_{2j}-n_{2j}\right)\left(\underline{b}_1\cdot\underline{b}_2\right)+\|\underline{b}_1\|^2\left(\Delta\tilde{\phi}_{2j}-n_{2j}\right)^2\end{aligned} \tag{4}$$

Note, the same inequality can be applied using sightlines $\underline{s}_1$ and $\underline{s}_2$:

$$\|\underline{b}_i\|^2\left[1-\left(\underline{s}_1\cdot\underline{s}_2\right)^2\right] > \left(\Delta\tilde{\phi}_{i1}-n_{i1}\right)^2 - 2\left(\Delta\tilde{\phi}_{i1}-n_{i1}\right)\left(\Delta\tilde{\phi}_{i2}-n_{i2}\right)\left(\underline{s}_1\cdot\underline{s}_2\right)+\left(\Delta\tilde{\phi}_{i2}-n_{i2}\right)^2 \tag{5}$$

If the integers have been properly resolved then it can be shown that Equation (4) reduces down to (in the noise free case)

$$\left[\left(A\underline{s}_j\right)\cdot\left(\underline{b}_1\times\underline{b}_2\right)\right]^2 > 0 \tag{6}$$

This means that $A\underline{s}_j$, $\underline{b}_1$ and $\underline{b}_2$ must not lie in the same plane. We need this condition to be able to extract attitude information outside of the $\underline{b}_1, \underline{b}_2$ plane. Note, Equation (6) is almost always satisfied if the integers pass the test in Equation (4). Equation (4) or (5) can be used to significantly reduce the search space, since only two baselines (or two sightlines) are considered at a time, as opposed to considering all three simultaneously.

The next step involves converting the sightlines into the body frame or converting the baselines into the reference frame.[12] For the former the algorithm begins by representing the $j$th sightline vector in the body frame, $A\underline{s}_j$, as the sum of two components. The first component $\hat{\underline{s}}_j$ is a function of the measured fractional phase measurements, and the second $\underline{c}_j$ depends on the unknown integer phase differences. This representation is accomplished by minimizing the following loss function

$$J\left(A\underline{s}_j\right)=\frac{1}{2}\sum_{i=1}^{M}\frac{1}{\varpi_{ij}^2}\left(\Delta\tilde{\phi}_{ij}-n_{ij}-\underline{b}_i^T A\underline{s}_j\right)^2 \quad \text{for } j=1,2,\ldots,N \tag{7}$$

where $M$ is the number of baselines and $N$ is the number of available sightlines. If at least three non-coplanar baselines exist, the minimization of Equation (7) is straightforward and leads to

$$A\underline{s}_j = \underline{\hat{s}}_j - \underline{c}_j \tag{8a}$$

$$\underline{\hat{s}}_j = B_j^{-1}\left[\sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2} \Delta\tilde{\phi}_{ij} \underline{b}_i\right] \tag{8b}$$

$$\underline{c}_j = B_j^{-1}\left[\sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2} n_{ij}\, \underline{b}_i\right] \tag{8c}$$

$$B_j = \sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2} \underline{b}_i \underline{b}_i^T \tag{8d}$$

Since the measurements are not perfect, Equation (8a) is replaced by the effective measurement model

$$\underline{\hat{s}}_j = A\,\underline{s}_j + \underline{c}_j + \underline{\varepsilon}_j \tag{9}$$

where $\underline{c}_j$ is a constant bias since the baselines are assumed constant, and $\underline{\varepsilon}_j$ is a zero-mean Gaussian process with covariance $R_j = B_j^{-1}$. This model is used for the actual attitude determination,[12] which we will not consider further in this paper.

The next step is to use an attitude-independent method to find the phase-bias vector $\underline{c}_j$ for each sightline, which gives all the sightlines in both the body frame and the reference frame. The explicit integer phases are not needed for this solution, but it is important to check that they are close to integer values, as mentioned in the Introduction. In the general case, the explicit integer phases can be found from the attitude solution. The three-baseline case ($M = 3$) is simpler, for in this case Equation (5c) can be inverted to give

$$n_{ij} = \underline{b}_i^T\, \underline{c}_j \tag{10}$$

With more than three baselines, however, Equation (8c) does not have a unique solution for $\underline{c}_j$, so the $M$ integer phases for sightline $\underline{s}_j$ cannot be found from $\underline{c}_j$ alone. We will consider the three-baseline case, which is the most common in practice. If more baselines are available, we are always free to select a three-baseline subset. Then, after the integer phases have been determined, a refined attitude estimate can be computed using all baselines (i.e., three baselines are sufficient to determine an attitude, which may then be used to resolve the integers corresponding to the other baselines).

To eliminate the dependence on the attitude, the orthogonality of $A$ and Equation (9) are used to give

$$\begin{aligned}\left\|\underline{s}_j\right\|^2 = \left\|A\,\underline{s}_j\right\|^2 &= \left\|\hat{\underline{s}}_j - \underline{c}_j - \underline{\varepsilon}_j\right\|^2 \\ &= \left\|\hat{\underline{s}}_j\right\|^2 - 2\,\hat{\underline{s}}_j \cdot \underline{c}_j + \left\|\underline{c}_j\right\|^2 - 2\left(\hat{\underline{s}}_j - \underline{c}_j\right)\cdot\underline{\varepsilon}_j + \left\|\underline{\varepsilon}_j\right\|^2\end{aligned} \tag{11}$$

Next, following Alonso and Shuster, the following effective measurement and noise are defined[13]

$$z_j \equiv \left\|\hat{\underline{s}}_j\right\|^2 - \left\|\underline{s}_j\right\|^2 \tag{12a}$$

$$v_j \equiv 2\left(\hat{\underline{s}}_j - \underline{c}_j\right)\cdot\underline{\varepsilon}_j - \left\|\underline{\varepsilon}_j\right\|^2 \tag{12b}$$

Then, the effective measurement model is

$$z_j = 2\,\hat{\underline{s}}_j \cdot \underline{c}_j - \left\|\underline{c}_j\right\|^2 + v_j \tag{13}$$

where $v_j$ is approximately Gaussian for small $\underline{\varepsilon}_j$ with mean and variance given by

$$\mu_j \equiv E\left\{v_j\right\} = -\text{trace}\left\{R_j\right\} \tag{14}$$

and

$$\sigma_j^2 \equiv E\left\{v_j^2\right\} - \mu_j^2 = 4\left(\hat{\underline{s}}_j - \underline{c}_j\right)^T R_j\left(\hat{\underline{s}}_j - \underline{c}_j\right) - \mu_j^2 \tag{15}$$

respectively. Equations (12)-(15) define an attitude-independent algorithm because they do not contain the attitude matrix $A$.

The negative-log-likelihood function for the bias is given by

$$J\left(\underline{c}_j\right) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\sigma_j^2(k)}\left[z_j(k) - 2\,\hat{\underline{s}}_j(k)\cdot\underline{c}_j + \left\|\underline{c}_j\right\|^2 - \mu_j(k)\right]^2 + \log\sigma_j^2(k) + \log 2\pi\right\} \tag{16}$$

where $L$ is the total number of measurement epochs, and the symbol $k$ denotes the variable at time $t_k$. The maximum-likelihood estimate for $\underline{c}_j$, denoted by $\underline{c}_j^*$, minimizes the negative-log-likelihood function, and satisfies

$$\left.\frac{\partial J\left(\underline{c}_j\right)}{\partial \underline{c}_j}\right|_{\underline{c}_j^*} = \underline{0} \tag{17}$$

The minimization of Equation (16) is not straightforward since the likelihood function is quartic in $\underline{c}_j$. A number of algorithms have been proposed for estimating the bias (see Ref. 13 for a survey). The simplest solution is obtained by scoring, which involves a Newton-Raphson iterative approach. Another approach avoids the minimization of a quartic loss function by using a "centered" estimate. A statistically correct centered estimate is also derived in Ref. 13. Furthermore, Alonso and Shuster show a complete solution of the statistically correct centered estimate that determines the exact maximum likelihood estimate $\underline{c}_j^*$. This involves using the statistically correct centered estimate as an initial estimate, and iterating on a correction term using a Gauss-Newton method. Although this extension to the statistically correct centered estimate can provide some improvements, this part is not deemed necessary for the GPS problem since the estimated quantity for $n_{ij}$ is rounded to the nearest integer.

In this paper another approach is used. As before, we consider the case for $M = 3$. Equations (8b) and (8c) are first re-written as

$$\hat{\underline{s}}_j = B_j^{-1}\,\Gamma_j\,\underline{\Phi}_j \tag{18a}$$

$$\underline{c}_j = B_j^{-1}\,\Gamma_j\,\underline{n}_j \tag{18b}$$

where

$$\Gamma_j \equiv \begin{bmatrix} \varpi_{1j}^{-2}\,\underline{b}_1 & \varpi_{1j}^{-2}\,\underline{b}_2 & \varpi_{1j}^{-2}\,\underline{b}_3 \end{bmatrix} \tag{19a}$$

$$\underline{n}_j \equiv \begin{bmatrix} n_{1j} \\ n_{2j} \\ n_{3j} \end{bmatrix} \tag{19b}$$

$$\underline{\Phi}_j \equiv \begin{bmatrix} \Delta\tilde{\phi}_{1j} \\ \Delta\tilde{\phi}_{2j} \\ \Delta\tilde{\phi}_{3j} \end{bmatrix} \tag{19c}$$

The loss function in Equation (16) can now be re-written (neglecting the term independent of $\underline{n}_j$):

$$J(\underline{n}_j) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\sigma_j^2(k)}\left[\left\|B_j^{-1}\Gamma_j\left(\underline{\Phi}_j(k)-\underline{n}_j\right)\right\|^2 - \left\|\underline{s}_j(k)\right\|^2 + \operatorname{trace}\left\{B_j^{-1}\right\}\right]^2 + \log\sigma_j^2(k)\right\} \tag{20}$$

with

$$\sigma_j^2(k) = \left(\underline{\Phi}_j(k)-\underline{n}_j\right)^T\Gamma_j^T B_j^{-3}\,\Gamma_j\left(\underline{\Phi}_j(k)-\underline{n}_j\right) - \operatorname{trace}^2\left\{B_j^{-1}\right\} \tag{21}$$

Equation (20) can now be used to directly determine the integers without pre-computing the sightline vector in the body frame. Equation (20) clearly indicates that the loss function involves a scalar check on the norm vector residuals (since $B_j^{-1}\Gamma_j\left(\underline{\Phi}_j-\underline{n}_j\right) = \hat{\underline{s}}_j-\underline{c}_j$).[14] In practice if $\underline{n}_j$ is real valued, then a sufficient amount of vehicle motion must occur in order to determine the minimum. This was the approach used in Ref. [11]. However, the solution in this paper involves checking the remaining integers that have passed the inequality condition in Equation (4). Since the solutions for the components of $\underline{n}_j$ are constrained to be integers, then it is more likely that a unique solution which minimizes Equation (21) can be determined with minimal vehicle motion.

The estimate error covariance can also be computed in order to insure that the determined integer is statistically correct. This can be shown to be given in the limit of infinitely large samples by

$$P_j = \left\{ \sum_{k=1}^{L} \frac{4}{\sigma_j^2(k)} \left[ \underline{\Phi}_j(k) - \underline{n}_j \right] \left[ \underline{\Phi}_j(k) - \underline{n}_j \right]^T \right\}^{-1} \tag{22}$$

Equation (22) can be used to develop an integrity check for the algorithm, using standard results on hypothesis testing.[15] For example, the computed integer can be shown to have only a 0.0013 probability of selecting the wrong integer when three times the square root of a diagonal element of $P_j$ is less than 1/2.

The case where 3 coplanar baselines can be determined by considering 3 non-coplanar sightlines. A batch solution for this case can be determined using a similar approach shown in Ref. 11; however, the statistically correct centered estimate approach is complex since the sightlines vary with time. Also, this requires that the same 3 sightlines are available long enough to determine a solution (which isn't always possible). The integer search approach presented here may alleviate these difficulties. The loss function for this case is given by

$$J(\underline{n}_i) = \frac{1}{2} \sum_{k=1}^{L} \left\{ \frac{1}{\sigma_i^2(k)} \left[ \left\| S_i^{-1}(k) \Gamma_i(k) \left( \underline{\Phi}_i(k) - \underline{n}_i \right) \right\|^2 - \left\| \underline{b}_i \right\|^2 + \text{trace}\left\{ S_i^{-1}(k) \right\} \right]^2 + \log \sigma_i^2(k) \right\} \tag{23}$$

where

$$\sigma_i^2(k) = \left( \underline{\Phi}_i(k) - \underline{n}_i \right)^T \Gamma_i^T(k) S_i^{-3}(k) \Gamma_i(k) \left( \underline{\Phi}_i(k) - \underline{n}_i \right) - \text{trace}^2\left\{ S_i^{-1}(k) \right\} \tag{24a}$$

$$\Gamma_i \equiv \left[ \varpi_{i1}^{-2}\, \underline{s}_1 \quad \varpi_{i2}^{-2}\, \underline{s}_2 \quad \varpi_{i3}^{-2}\, \underline{s}_3 \right] \tag{24b}$$

$$\underline{n}_i \equiv \begin{bmatrix} n_{i1} \\ n_{i2} \\ n_{i3} \end{bmatrix} \tag{24c}$$

$$\underline{\Phi}_i \equiv \begin{bmatrix} \Delta\tilde{\phi}_{i1} \\ \Delta\tilde{\phi}_{i2} \\ \Delta\tilde{\phi}_{i3} \end{bmatrix} \tag{24d}$$

$$S_i = \varpi_{i1}^{-2}\, \underline{s}_1 \underline{s}_1^T + \varpi_{i2}^{-2}\, \underline{s}_2 \underline{s}_2^T + \varpi_{i3}^{-2}\, \underline{s}_3 \underline{s}_3^T \tag{24e}$$

For this case both $\Gamma$ and $S$ vary with time. The estimate error covariance for this case is given by

$$P_i = \left\{ \sum_{k=1}^{L} \frac{4}{\sigma_i^2(k)} \left[ \underline{\Phi}_i(k) - \underline{n}_i \right] \left[ \underline{\Phi}_i(k) - \underline{n}_i \right]^T \right\}^{-1} \tag{25}$$

Once again, this can be used to develop an integrity check for the algorithm.

There are many advantages of the new algorithm. First, the algorithm is fully autonomous (i.e., it requires no a-priori information such as an a-priori attitude guess). Second, it can be used to determine the integers when 3 coplanar baselines exist. Third, the required search space can be significantly reduced using Equation (4) or (5). Finally, the integers for other sightlines (or baselines) can be easily resolved by calling the same subroutine. For these reasons, the new algorithm provides an attractive approach to resolve the integers.

## Integer Ambiguity Simulation Results

A simulation of a CRV escape maneuver has been undertaken. The overall goal of the simulation study is to evaluate how fast the integers can be resolved, and to assess the reliability of the determine solutions. The simulated CRV has four GPS antennas that form three baselines (this is typical for most GPS attitude determination systems). The antenna separation distances are 0.61 m, 1.12 m, and 1.07 m, respectively. One antenna (in baseline 3) is located 0.23 m out of plane (below) the other three antennas. On the CRV, the antennas are assumed to be mounted on pedestals to minimize signal reflections and multipath. For the simulation, multipath errors are introduced using a simple Markov-process with time constant of 5 seconds and standard deviation of 0.026 wavelengths.[16] The baseline vectors in wavelengths are given by

$$\underline{b}_1 = \begin{bmatrix} 2.75 \\ 1.64 \\ -0.12 \end{bmatrix}, \quad \underline{b}_2 = \begin{bmatrix} 0.00 \\ 6.28 \\ -0.17 \end{bmatrix}, \quad \underline{b}_3 = \begin{bmatrix} -3.93 \\ 3.93 \\ -1.23 \end{bmatrix} \tag{26}$$

The raw measurements are processed at 1 Hz over a forty minute simulation. During the simulated run, a minimum of three visible pseudolites are in sight at all times. However, resolution of the ambiguous integers for the phase measurements from any specific pseudolite requires that it remain in view continuously until the sequential algorithm converges. In practice, all available sightlines should be processed, since attitude determination requires the integers to be resolved for two pseudolites simultaneously. The simulation contains a number of spans when sightlines to two specific pseudolites are continuously available for the ambiguity resolution algorithm.

Since the baselines are non-coplanar, Equations (4) and (20) will used to determine the integers. For the simulation the following integer ambiguities are introduced:

$$\underline{n}_1 = \begin{bmatrix} -6 \\ 1 \\ 3 \end{bmatrix}, \quad \underline{n}_2 = \begin{bmatrix} 5 \\ -8 \\ -2 \end{bmatrix} \tag{27}$$

From the baseline geometry the integers are bounded from –8 to 8. If a full search is implemented using Equation (20) solely this would require 8192 searches (4096 for each sightline). Equation (4) requires a total of only 1024 searches. The integers at the initial time that passed the inequality in Equation (4) are shown in Table 3.

Table 3 clearly shows how Equation (4) can be used to dramatically reduce the search space. Only 9 remaining searches need to be used in Equation (20). Since the computational load to do this search is extremely low, a search can be implemented every few seconds or so. For this simulation the cost function in Equation (20) is summed over time and has been checked every 5 seconds. The stopping condition is given when three times the square root of every diagonal element of $P_j$ is less than 1/2. For this case, this occurred in 15 seconds for each sightline. The associated cost for each integer set after 15 seconds is also shown in Table 3. Clearly, the correct integers have been found. Also, another simulation using Equations (5) and (23) has been performed in order to investigate the performance using 3 non-coplanar sightlines. Results indicate that this approach give comparable results to the results shown in Table 3. Finally, 100 Mote Carlo type simulations have been executed in order to assess the reliability of the new algorithm. In every case the integers were determined correctly within 15 seconds.

**Table 3 Remaining Integers and Associated Costs**

| | Integers | Cost |
|---|---|---|
| First Baseline | -6 1 3 | 5.3 |
| | -3 5 3 | 10.2 |
| | -3 7 3 | 13.6 |
| | -1 8 2 | 145.2 |
| Second Baseline | 5 -8 -2 | 5.7 |
| | 7 -6 -3 | 11.4 |
| | 7 -5 -3 | 13.8 |
| | 8 -8 -8 | 26.3 |
| | 8 -7 -7 | 94.6 |

## Conclusions

In this paper, the use of an autonomous (independent of ISS power or data systems) set of radio transmitters (pseudolites) is explored for relative vehicle-to-vehicle attitude determination. Two vital issues were addressed: (1) the effect of non-planar wavefronts due to making measurements from the near-field of pseudolites, and (2) the resolution of the integer ambiguities due to the need for phase difference measurements. Simulation results show that the attitude accuracy for a simple estimator degrades in the presence of non-planar wavefronts. Still, the potential exists for attitude determination within few-degree accuracy per axis. A new algorithm was developed for GPS integer ambiguity resolution. The algorithm uses the best qualities of both instantaneous and motion-based techniques. It uses an instantaneous approach to substantially reduce the search space, and then uses a batch-type loss function to resolve the remaining possible integers. The new algorithm has several advantages over previously existing algorithms. First, the algorithm is attitude independent so that no a-priori attitude estimate (or assumed vehicle motion) is required. Second, a suitable integrity check can be used to determine the correct values. Finally, it can resolve the integers even when coplanar baselines exist. The algorithm was tested using a simulation of a typical escape maneuver for the Crew Return Vehicle. Results indicated that the new algorithm provides a viable and attractive means to effectively resolve the integer ambiguities. Results for these studies clearly show that pseudolites can provide an effective means for relative attitude determination.

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