ATTITUDE DETERMINATION USING COMBINED GPS AND THREE-AXIS MAGNETOMETER DATA

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Both GPS and three-axis magnetometer (TAM) measurements can be combined using an optimal filter to provide robust attitude determination of spacecraft. The main advantage of this approach is that the TAM attitude solution is always available throughout the orbit, and may be used to estimate the attitude when GPS outages and anomalies occur (such as cycle slips). A more fundamental utilization of the combined sensors will be explored in this paper. The crucial aspect in GPS attitude determination is the resolution of the integer ambiguities. Several approaches have been investigated to resolve these integers. A new motion-based algorithm for GPS integer ambiguity resolution has been recently derived which converts the reference GPS sightline vectors into body frame vectors. This is accomplished by an optimal vectorized transformation of the phase difference measurements. The result of this transformation leads to the conversion of the integer ambiguities to vectorized biases, having the form identical to a TAM bias problem. Since the GPS integer resolution problem has been converted to a magnetometer-bias problem, if a calibrated TAM is used in conjunction with the GPS measurements then the integers can be resolved very quickly and reliably. Simulation results will be provided to show the usefulness of this approach.

Introduction

The utilization of phase difference measurements from Global Positioning System (GPS) receivers provides a novel approach for three-axis attitude determination and/or estimation. These measurements have been successfully used to determine the attitude of air-based,[1] space-based,[2,3] and sea-based[4] vehicles. Since phase differences are used, the correct number of integer wavelengths between a given pair of antennae must be found. The determination of the integer ambiguities can either be accomplished by using "static" (motionless) or "dynamic" (motion-based) techniques. The ambiguities essentially act as integer biases to the phase difference measurements. Once the integer ambiguities are resolved, then the attitude determination problem can be solved.[5]

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The static method finds a solution that minimizes the error residual at a specific time by searching through an exhaustive list of all possible integers and rejecting classes of solutions when the residual becomes too large.[6] Refinements can be made to the solution by restricting the search space with knowledge of a-priori information, such as the maximum tilt the baseline should encounter.[7] Static methods generally rely on solving a set of Diophantine equations.[8] The appeal of these methods is that they provide an "instantaneous" attitude solution, limited only by computation time, and are well suited for the short baselines. However, the minimum residual does not guarantee a correct solution in the presence of noise.[1] In fact, it is possible that static methods can report a wrong solution as valid, especially when some of the calibration information, such as line bias, is incorrect. This lack of integrity can cause significant problems if the sensor output is used to control a high bandwidth actuator, such as gas jets on a spacecraft. Another consideration is that static methods sometime require that the antenna array must be within a defined angle (typically 30 degrees) of a reference attitude, which is often true for ground-based applications, but is less likely for space-based applications. Also, structural flexibility in the baselines may lead to erroneous solutions. All of the aforementioned limitations imply that static methods, while attractive because of their fast solutions, are not totally acceptable for general purpose applications.

The other technique for resolving integer ambiguities involves collecting data for a given period of time and performing a batch solution, in which the integer terms remain constant over the collection period. This technique relies on the fact that a certain amount of motion has occurred during the data collection, either from vehicle body rotation or GPS line-of-sight motion. The main disadvantage of this technique, compared to static approaches, is that it takes time for the motion to occur, which may be on the order of several minutes. Another consideration is that a potentially significant amount of memory is required for the storage of the batch data collection. But, motion-based techniques also have significant advantages over static methods. Most importantly, motion-based techniques are inherently high integrity methods because there are numerous checks that can be implemented into the solution before it is accepted. These include using statistical checks applied to error residuals, matrix condition number checks, and using the closeness of the computed floating-point "integers" to actual integers as a check. The probability of an erroneous solution being reported as valid can be made as small as desired by appropriately setting the thresholds on these integrity checks. For these reasons, motion-based techniques have been more widely used for on-board applications.

For most spacecraft that are orbiting beneath the GPS constellation, e.g. in Low Earth Orbit (LEO), a readily available attitude sensor capability is a Three-Axis Magnetometer (TAM). A TAM measures the geomagnetic field in body coordinates which is compared to a reference model of the Earth's magnetic field, using the position of the spacecraft, for attitude determination. Although the TAM provides three scalar measurements, only two directions of attitude can be determined at any given point in time (i.e. all but the rotation about the field magnitude). These attitude sensors have many advantages including: 1) TAMs are relatively inexpensive and reliable, and are already present in most spacecraft for magnetic actuator control, 2) the Earth's magnetic field is always in the field-of-view, and 3) three-axis attitude determination is possible with some knowledge of the spacecraft dynamics (usually via gyroscopic measurements or dynamic models), using information from past measurements in batch or filter solutions.[9] Recently, GPS and TAM sensor data were combined in a filter with a dynamic model to estimate the attitude of the REX-II spacecraft.[2,10] The combined sensor output filter is considered to be more practical for sub-degree controller performance because it provides an acceptable attitude measurement even during periods of GPS attitude sensor outage, which has been shown to occur routinely with current GPS receiver hardware during normal spacecraft operations.

In this paper a more fundamental use of GPS phase measurements combined with TAMs is given. A new motion-based algorithm has been recently derived,[11] which has been shown to have significant advantages over prior methods, including: 1) it resolves the integer ambiguities without any a-priori attitude knowledge, 2) it requires less computational effort, since large matrix inverses are not needed, and 3) it is non-iterative. A disadvantage of the new algorithm is that it requires at least three non-coplanar baselines. The first step of this algorithm involves determining the reference GPS sightline vectors into body frame vectors. This is accomplished by an optimal vectorized

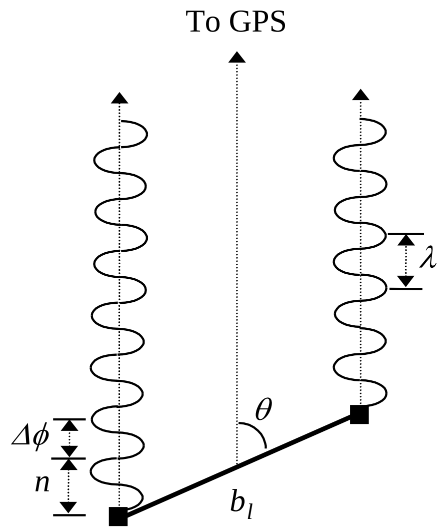

Figure 1: GPS Wavelength and Wavefront Angle

transformation of the phase difference measurements. The result of this transformation leads to the conversion of the integer ambiguities to vectorized biases. This essentially converts the problem to a magnetometer-bias determination problem, for which optimal and efficient solutions exist.[15] Using a TAM as another effective measurement with the GPS allows for faster integer ambiguity resolution since only two baselines are required (as will be demonstrated), as well as providing a backup system during GPS outages.

The organization of this paper proceeds as follows. First, a review of the GPS and TAM sensor models is shown. Then, the conversion of the GPS sightline vector into the body frame is summarized. Next, GPS phase difference measurements are combined with TAM measurements in order to provide a fast and reliable approach for integer ambiguity resolution. Then, an optimal attitude determination algorithm is reviewed. Finally, the approach is tested using a simulated spacecraft maneuver.

Sensor Models

In this section a brief background of the GPS phase difference measurement and TAM measurement is shown. The GPS constellation of spacecraft was developed for accurate navigation information of land-based, air, and spacecraft user systems. Spacecraft applications initially involved obtaining accurate orbit information and accurate time-tagging of spacecraft operations. However, attitude determination of vehicles, such as spacecraft or aircraft, has gained much attention. The main measurement used for GPS attitude determination is the phase difference of the signal received from two antennae separated by a baseline. The principle of the wavefront angle and wavelength, which are used to develop a phase difference, is illustrated in Figure 1. The phase difference measurement is obtained by

$$b_l \cos\theta = \lambda(\Delta\phi - n) \tag{1}$$

where b_l is the baseline length (in cm), θ is the angle between the baseline and the line-of-sight to the GPS spacecraft, n is the number of integer wavelengths between two antennae, $\Delta\phi$ is the phase difference (in cycles), and λ is the wavelength (in cm) of the GPS signal. The two GPS frequency carriers are L1 at 1575.42 MHz and L2 at 1227.6 MHz. As of this writing, non-military applications generally use the L1 frequency. The phase difference can be expressed by

$$\Delta\phi = \mathbf{b}^T A\mathbf{s} + n \tag{2}$$

where $\mathbf{s} \in \Re^3$ is the normalized sightline vector to the GPS spacecraft in a reference frame, typically Earth-Centered-Earth-Fixed (ECEF), $\mathbf{b} \in \Re^3$ is the baseline vector (in wavelengths), which is the relative position vector from one antenna to another, and $A \in \Re^{3\times3}$ is the attitude matrix that maps the reference frame to the body frame, which is an orthogonal matrix with determinant +1 (i.e. $A^T A = I_{3\times3}$). The measurement model is given by

$$\Delta\tilde{\phi}_{ij} = \mathbf{b}_i^T A \mathbf{s}_j + n_{ij} + v_{ij} \tag{3}$$

where $\Delta\tilde{\phi}_{ij}$ denotes the phase difference measurement for the i^{th} baseline and j^{th} sightline, and v_{ij} represents a zero-mean Gaussian measurement error with standard deviation σ_{ij} which is $0.5\text{cm}/\lambda = 0.026$ wavelengths for typical phase noise.[1] At each epoch it is assumed that M baselines and N sightlines exist. The minimum number of baselines and sightlines required to determine the attitude within an ambiguity (arising from an intersection of two cones) is two baselines and two sightlines.[12] A unique attitude solution exists if any more number of baselines or sightlines are available at a given epoch.

A TAM measures three components of the Earth's magnetic field. The measurement model is assumed to be of the form given by

$$\tilde{\mathbf{B}}_B = A\mathbf{B}_I + \boldsymbol{v} \tag{4}$$

where $\tilde{\mathbf{B}}_B \in \Re^3$ is the magnetic field measurement in the space body coordinate system, $\mathbf{B}_I \in \Re^3$ is the known magnetic field vector in inertial coordinates, and $\boldsymbol{v}$ is assumed to be zero-mean Gaussian measurement error with standard deviation $\sigma_m I_{3\times3}$. The actual error is not Gaussian since the dominate noise source comes from errors in the assumed magnetic field model (which are typically nonlinear effects).[13] For near-Earth orbits, the error in the magnetic field model may vary from 0.5 degrees near the equator to 3 degrees near the magnetic poles, where erratic auroral currents play a large role.[14] Typically the reference frame of the magnetic field is different than the GPS reference frame; however, the transformation to a common frame is easily accomplished since the position of the spacecraft is assumed to be known.

Integer Ambiguity Resolution

In this section an attitude-independent algorithm to resolve the integer ambiguities is presented. The algorithm is presented assuming that three non-coplanar baselines exist. The first step involves a conversion of the sightline vectors into the body frame. This in essence converts the integer ambiguity problem into a form exactly as a magnetometer-bias problem. Then, the integers are resolved using a batch solution. Finally, TAM measurements are combined with GPS phase difference measurements in order to resolve the integers using only two baselines.

GPS Measurements Only

The attitude-independent algorithm using GPS phase difference measurements begins by determining the j^{th} sightline vector in the body frame, $A\mathbf{s}_j$, as the sum of two components. The first component $\hat{\mathbf{s}}_j$ is a function of the measured fractional phase measurements, and the second $\mathbf{c}_j$ depends on the unknown integer phase differences. This representation is accomplished by minimizing the following loss function:

$$J(A\mathbf{s}_j) = \frac{1}{2}\sum_{i=1}^{M} \sigma_{ij}^{-2} \left(\Delta\tilde{\phi}_{ij} - n_{ij} - \mathbf{b}_i^T A\mathbf{s}_j\right)^2 \quad \text{for } j = 1, 2, \ldots, N \tag{5}$$

If at least three non-coplanar baselines exist, the minimization of Eq. (5) is straightforward and leads to

$$A\mathbf{s}_j = \hat{\mathbf{s}}_j - \mathbf{c}_j \tag{6a}$$

$$\hat{\mathbf{s}}_j = B_j^{-1}\left[\sum_{i=1}^{M} \sigma_{ij}^{-2} \Delta\tilde{\phi}_{ij}\, \mathbf{b}_i\right] \tag{6b}$$

$$\mathbf{c}_j = B_j^{-1}\left[\sum_{i=1}^{M} \sigma_{ij}^{-2} n_{ij}\, \mathbf{b}_i\right] \tag{6c}$$

$$B_j = \sum_{i=1}^{M} \sigma_{ij}^{-2} \mathbf{b}_i \mathbf{b}_i^T \tag{6d}$$

Since the measurements are not perfect, Eq. (6a) is replaced by the following measurement model

$$\hat{\mathbf{s}}_j = A\mathbf{s}_j + \mathbf{c}_j + \boldsymbol{\varepsilon}_j \tag{7}$$

where $\mathbf{c}_j$ is a constant bias since the baselines are assumed constant, and $\boldsymbol{\varepsilon}_j$ is a zero-mean Gaussian noise process with covariance $R_j = B_j^{-1}$. This covariance exists only if three non-coplanar baseline vectors exist.

The next step is to use an attitude-independent method to find the phase-bias vector $\mathbf{c}_j$ for each sightline, which gives all the sightlines in both the body frame and the reference frame. The explicit integer phases are not needed for this solution, but it is important to check that they are close to integer values. In the general case, the explicit integer phases can be found from the attitude solution. The three-baseline case ($M = 3$) is simpler, for in this case Eq. (6c) can be inverted to give

$$n_{ij} = \mathbf{b}_i^T \mathbf{c}_j \tag{8}$$

With more than three baselines, however, Eq. (6a) does not have a unique solution for $\mathbf{c}_j$, so the M integer phases for sightline $\mathbf{s}_j$ cannot be found from $\mathbf{c}_j$ alone. We will consider the three-baseline case, which is the most common in practice. If more baselines are available, we are always free to select a three-baseline subset. Then, after the integer phases have been determined, a refined attitude estimate can be computed using all baselines (i.e. three baselines are sufficient to determine an attitude, which may then be used to resolve the integers corresponding to the other baselines).

To eliminate the dependence on the attitude, the orthogonality of A and Eq. (7) are used to give

$$\begin{aligned} \|\mathbf{s}_j\|^2 = \|A\mathbf{s}_j\|^2 &= \|\hat{\mathbf{s}}_j - \mathbf{c}_j - \boldsymbol{\varepsilon}_j\|^2 \\ &= \|\hat{\mathbf{s}}_j\|^2 - 2\hat{\mathbf{s}}_j \cdot \mathbf{c}_j + \|\mathbf{c}_j\|^2 - 2(\hat{\mathbf{s}}_j - \mathbf{c}_j)\cdot \boldsymbol{\varepsilon}_j + \|\boldsymbol{\varepsilon}_j\|^2 \end{aligned} \tag{9}$$

Next, the following effective measurement and noise are defined:

$$z_j \equiv \|\hat{\mathbf{s}}_j\|^2 - \|\mathbf{s}_j\|^2 \tag{10a}$$

$$\nu_j \equiv 2(\hat{\mathbf{s}}_j - \mathbf{c}_j)\cdot \boldsymbol{\varepsilon}_j - \|\boldsymbol{\varepsilon}_j\|^2 \tag{10b}$$

Then, the effective measurement model is

$$z_j = 2\hat{\mathbf{s}}_j \cdot \mathbf{c}_j - \|\mathbf{c}_j\|^2 + \nu_j \tag{11}$$

Alonso and Shuster[15] showed that ν_j is approximately Gaussian for small $\boldsymbol{\varepsilon}_j$ with mean given by

$$\mu_j \equiv E\left\{\nu_j\right\} = -\mathrm{trace}\left\{R_j\right\} \tag{12}$$

and variance given by

$$\varpi_j^2 \equiv E\left\{\nu_j^2\right\} - \mu_j^2 = 4(\hat{\mathbf{s}}_j - \mathbf{c}_j)^T R_j (\hat{\mathbf{s}}_j - \mathbf{c}_j) - \mu_j^2 \tag{13}$$

Equations (10)-(13) define an attitude-independent measurement model because they do not contain the attitude matrix A.

The negative-log-likelihood function for the bias is given by

$$J(\mathbf{c}_j) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\varpi_j^2(k)}\left[z_j(k) - 2\hat{\mathbf{s}}_j(k)\cdot\mathbf{c}_j + \|\mathbf{c}_j\|^2 - \mu_j(k)\right]^2 + \ln\varpi_j^2(k) + \ln 2\pi\right\} \tag{14}$$

where L is the total number of measurement epochs, and the symbol k denotes that the variable at time t_k. The maximum-likelihood estimate for $\mathbf{c}_j$, denoted by $\mathbf{c}_j^*$, minimizes the negative-log-likelihood function, and satisfies

$$\left.\frac{\partial J(\mathbf{c}_j)}{\partial \mathbf{c}_j}\right|_{\mathbf{c}_j^*} = \mathbf{0} \tag{15}$$

The minimization of Eq. (14) is not straightforward since the likelihood function is quartic in $\mathbf{c}_j$. A number of algorithms have been proposed for estimating the bias (see Ref. [15] for a survey). The simplest solution is obtained by scoring, which involves a Newton-Raphson iterative approach. Another approach avoids the minimization of a quartic loss function by using a "centered" estimate. A statistically correct centered estimate is also derived in Ref. [15]. Furthermore, Alonso and Shuster show a complete solution of the statistically correct centered estimate that determines the exact maximum likelihood estimate $\mathbf{c}_j^*$. This involves using the statistically correct centered estimate as an initial estimate, and iterating on a correction term using a Gauss-Newton method. Although this extension to the statistically correct centered estimate can provide some improvements, this part is not deemed necessary for the GPS problem since the estimated quantity for n_{ij} is rounded to the nearest integer.

The loss function in Eq. (14) can also be rewritten as a function of the unknown integers explicitly. As before, we consider the case for $M = 3$. Equations (6b) and (6c) are first rewritten as

$$\hat{\mathbf{s}}_j = B_j^{-1}\Gamma_j\mathbf{\Phi}_j \tag{16a}$$

$$\mathbf{c}_j = B_j^{-1}\Gamma_j\mathbf{n}_j \tag{16b}$$

where

$$\Gamma_j \equiv \left[\sigma_{1j}^{-2}\mathbf{b}_1 \quad \sigma_{2j}^{-2}\mathbf{b}_2 \quad \sigma_{3j}^{-2}\mathbf{b}_3\right] \tag{17a}$$

$$\mathbf{n}_j \equiv \left[n_{1j} \;\; n_{2j} \;\; n_{3j}\right]^T \tag{17b}$$

$$\mathbf{\Phi}_j \equiv \left[\Delta\tilde{\phi}_{1j} \;\; \Delta\tilde{\phi}_{2j} \;\; \Delta\tilde{\phi}_{3j}\right]^T \tag{17c}$$

The loss function in Eq. (14) can now be rewritten as

$$J(\mathbf{n}_j) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\varpi_j^2(k)}\left[\|B_j^{-1}\Gamma_j\left(\mathbf{\Phi}_j(k) - \mathbf{n}_j\right)\|^2 - \|\mathbf{s}_j(k)\|^2 + \text{trace}\left\{B_j^{-1}\right\}\right]^2 + \ln\varpi_j^2(k) + \ln 2\pi\right\} \tag{18}$$

with

$$\varpi_j^2(k) = \left(\mathbf{\Phi}_j(k) - \mathbf{n}_j\right)^T\Gamma_j^T B_j^{-3}\Gamma_j\left(\mathbf{\Phi}_j(k) - \mathbf{n}_j\right) - \text{trace}^2\left\{B_j^{-1}\right\} \tag{19}$$

Equation (18) can now be used to directly determine the integers without pre-computing the sightline vector in the body frame. Equation (18) clearly indicates that the loss function involves a scalar check on the norm vector residuals (since $B_j^{-1}\Gamma_j\left(\mathbf{\Phi}_j(k) - \mathbf{n}_j\right) = \hat{\mathbf{s}}_j - \mathbf{c}_j$). In practice if $\mathbf{n}_j$ is real valued, then a sufficient amount of vehicle motion must occur in order to determine the minimum. This was the approach used in Ref. [11].

The estimate error covariance for the integer vector $\mathbf{n}_j$ can also be computed in order to insure that the determined integer is statistically correct. This can be shown to be given in the limit of infinitely large samples by

$$P_j = \left\{\sum_{k=1}^{L}\frac{4}{\varpi_j^2(k)}\left[\mathbf{\Phi}_j(k) - \mathbf{n}_j\right]\left[\mathbf{\Phi}_j(k) - \mathbf{n}_j\right]^T\right\}^{-1} \tag{20}$$

Equation (20) can be used to develop an integrity check for the algorithm, using standard results on hypothesis testing.[16] For example, the computed integer can be shown to have only a 0.0013 probability of selecting the wrong integer when three times the square root of a diagonal element of is less than 1/2.

Combined GPS and Magnetometer

For the combined GPS/TAM system we only assume a two baseline case, $\mathbf{b}_1$ and $\mathbf{b}_2$, in the GPS with corresponding phase difference measurements $\Delta\tilde{\phi}_{1j}$ and $\Delta\tilde{\phi}_{2j}$. Therefore, for each sightline we need only to solve for two integers. We also assume that the TAM has been properly calibrated so that its bias is eliminated from the measurement. Now, we use the TAM measurement as a "pseudo-phase difference" for the third GPS measurement. This pseudo-phase difference is given by the following:

$$\Delta\tilde{\phi}_{3j} \equiv \mathbf{B}_I^T \mathbf{s}_j \tag{21}$$

Substituting Eq. (4) for $\mathbf{B}_I$ into Eq. (21) gives

$$\Delta\tilde{\phi}_{3j} = \tilde{\mathbf{B}}_B^T A \mathbf{s}_j + \Delta v_j \tag{22}$$

where $\Delta v_j = -\mathbf{v}^T A \mathbf{s}_j$. Since the errors in the TAM measurements are assumed to be isotropic, the standard deviation of Δv_j is given $\sigma_{3j} \equiv \sigma_m$. Therefore the body magnetic field measurement $\tilde{\mathbf{B}}_B$ is treated as another "baseline" in the third phase difference observation, so that $\mathbf{b}_3 \equiv \tilde{\mathbf{B}}_B$.

The advantages of using a combined GPS/TAM system now become clear. Since the TAM has no bias, the associated integer in the pseudo-phase difference measurement (22) is zero (i.e. $n_{3j} = 0$). Therefore, we only need to solve for two integers n_{1j} and n_{2j}. The matrix B_j in Eq. (6d) is singular only if the TAM body measurement is in the same plane as $\mathbf{b}_1$ and $\mathbf{b}_2$, which is unlikely in most applications. The solution for the two integers can be now be found quickly be performing a check on all possible integer combinations to minimize Eq. (18). This signifies a substantial savings in the required computations. For example, consider the GPS-only case without TAM measurements. Let the baseline magnitudes in cycles (rounded down to the nearest integer) be given by l_1, l_2, and l_3. In order to determine the 3 integers for each sightline the required search space is given by $(2l_1+1)(2l_2+1)(2l_3+1)$, where the plus 1 term counts the 0 integer. But with the TAM measurement the search space is given by $(2l_1+1)(2l_2+1)$. This can, for long baselines, provide a dramatic reduction of at least an order of magnitude in the required search space.

The complete algorithm is as follows. For the first available sightline, using two baselines and the TAM pseudo-measurement perform a search on the possible integer pairs, evaluating the loss function in Eq. (18) (the integer n_{3j} is always zero). Choose the integer pair that minimizes Eq. (18). Evaluate the covariance in Eq. (20) in order to establish an integrity check. The 3-3 element of P_j should be very small if the TAM is well-calibrated. The 1-1 and 2-2 elements can be used to determine 3σ bounds on the found integers (as mentioned previously). A nonlinear least-squares solution can also be implemented using the found integers as an initial guess. This is useful to check that the least-squares solution is close to the found integers (i.e. the least-squares solution gives real values which should be close to the integers found from the search approach). Finally, integers for other baseline pairs can also be checked to see if common integer solutions match using identical baselines. For example, say that $\mathbf{b}_1$ and $\mathbf{b}_2$ are first used in the integer search algorithm; then by performing a search for the integers associated with baselines $\mathbf{b}_1$ and $\mathbf{b}_3$, the solution of the integer associated with baseline $\mathbf{b}_1$ can be checked to make sure it matches with the previous solution. This adds another integrity check in the algorithm. The entire process is repeated for the second available sightline.

Attitude Determination

Once all integers for the 2 baselines and 2 sightlines have been resolved, then attitude determination using the GPS and TAM measurements is possible. For attitude determination, the ALLEGRO[5] algorithm is used. A brief review of this algorithm is now presented. The ALLEGRO algorithm uses measurements at time t_{k+1} and previous estimates at time t_k to determine the body angular velocity of the vehicle. This angular velocity is then used to propagate a simple kinematics model from time t_{k+1} to time t_k.

The attitude matrix A in Eq. (2) is parameterized by the quaternion, defined as[17]

$$\mathbf{q} \equiv \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \tag{23}$$

with

$$\mathbf{q}_{13} \equiv \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T = \mathbf{e}\sin(\psi/2) \tag{24a}$$

$$q_4 = \cos(\psi/2) \tag{24b}$$

where $\mathbf{e}$ is a unit vector corresponding to the axis of rotation and ψ is the angle of rotation. The quaternion satisfies a single constraint given by $\mathbf{q}^T\mathbf{q} = 1$. The attitude matrix is related to the quaternion by

$$A(\mathbf{q}) = \Xi^T(\mathbf{q})\Psi(\mathbf{q}) \tag{25}$$

with

$$\Xi(\mathbf{q}) \equiv \begin{bmatrix} q_4 I_{3\times 3} + [\mathbf{q}_{13}\times] \\ -\mathbf{q}_{13}^T \end{bmatrix} \tag{26a}$$

$$\Psi(\mathbf{q}) \equiv \begin{bmatrix} q_4 I_{3\times 3} - [\mathbf{q}_{13}\times] \\ -\mathbf{q}_{13}^T \end{bmatrix} \tag{26b}$$

The 3×3 matrix $[\mathbf{q}_{13}\times]$ is referred to as a cross-product matrix because $\mathbf{a} \times \mathbf{b} = [\mathbf{a}\times]\,\mathbf{b}$, with

$$[\mathbf{a}\times] \equiv \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{27}$$

Using the available GPS measurements and TAM pseudo-measurement, an estimate for the angular velocity ($\boldsymbol{\omega}$) is given by

$$\begin{aligned} \boldsymbol{\omega}(t_k) = -\frac{1}{\Delta t}&\left\{\sum_{i=1}^{M}\sum_{j=1}^{N}\sigma_{ij}^{-2}\left[A\mathbf{s}_j^{\Delta}\times\right]\mathbf{b}_i\mathbf{b}_i^T\left[A\mathbf{s}_j^{\Delta}\times\right]^T\right\}^{-1} \\ &\times\sum_{i=1}^{M}\sum_{j=1}^{N}\sigma_{ij}^{-2}\left[A\mathbf{s}_j^{\Delta}\times\right]\mathbf{b}_i\left(\Delta\tilde{\phi}_{ij}^{\Delta} - \mathbf{b}_i^T A\mathbf{s}_j^{\Delta}\right) \end{aligned} \tag{28}$$

where the superscript Δ denotes that the quantity is measured at time t_{k+1} (all other quantities are at time t_k), and Δt is the sampling interval. The determined quaternion can be found by using a discrete-time propagation, given by

$$\hat{\mathbf{q}}(t_{k+1}) = [\chi(t_k)I_{4\times 4} + \mu(t_k)\Omega(t_k)]\hat{\mathbf{q}}(t_k) \tag{29}$$

where

$$\chi(t_k) = \cos\left(\frac{1}{2}\|\boldsymbol{\omega}(t_k)\|\Delta t\right) \tag{30a}$$

$$\mu(t_k) = \sin\left(\frac{1}{2}\|\boldsymbol{\omega}(t_k)\|\Delta t\right) \tag{30b}$$

$$\boldsymbol{\rho}(t_k) = \boldsymbol{\omega}(t_k)/\|\boldsymbol{\omega}(t_k)\| \tag{30c}$$

$$\Omega(t_k) = \begin{bmatrix} -[\boldsymbol{\rho}(t_k)\times] & \boldsymbol{\rho}(t_k) \\ -\boldsymbol{\rho}^T(t_k) & 0 \end{bmatrix} \tag{30d}$$

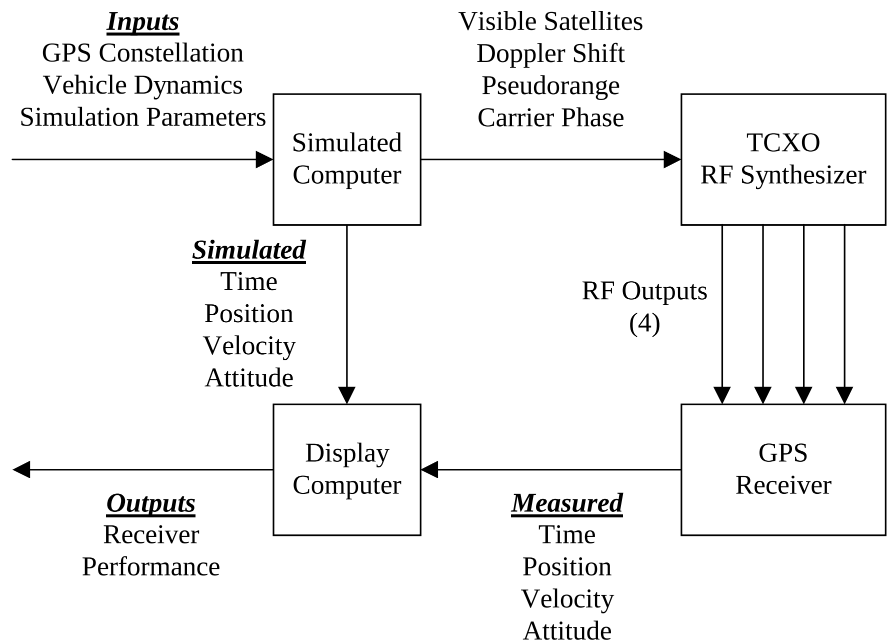

Figure 2: Hardware Simulation Block Diagram

For practical applications the sampling interval should be well below Nyquist's limit.[18] Equations (28)-(29) are used to determine the attitude at time t_{k+1} and previous estimates at time t_k.

The attitude error covariance at time t_{k+1} is given by

$$P(t_{k+1}) = \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \sigma_{ij}^{-2} \left[A\mathbf{s}_j^{\Delta} \times \right] \mathbf{b}_i \mathbf{b}_i^T \left[A\mathbf{s}_j^{\Delta} \times \right]^T \right\}^{-1} \tag{31}$$

The diagonal elements of this expression give the small angle attitude errors for roll, pitch, and yaw, irregardless of the attitude rotation sequence. Therefore Eq. (31) can be used to assess the expected errors in the attitude solutions (i.e. the errors between the actual attitude and the determined attitude).

Hardware Simulation

A hardware simulation of a typical spacecraft attitude determination application was undertaken to demonstrate the performance of the combined GPS/TAM approach. A LEO spacecraft was simulated, which nominally points at the center of the Earth. For this simulation, a Northern Telecom 40 channel, 4 RF output STR 2760 unit was used to generate the GPS signals that would be received at a user specified location and velocity. The signals are then provided directly (i.e. they are not actually radiated) to a GPS receiver that has been equipped with software tracking algorithms that allow it operate in space (see Figure 2).

The receiver that was used was a Trimble TANS Vector; which is a 6 channel, 4 RF input multiplexing receiver that performs 3-axis attitude determination using GPS carrier phase and line-of-sight measurements. This receiver was modified in software at Stanford University and NASA's Goddard Space Flight Center (GSFC) to allow it to operate in space. This receiver model has flown and operated successfully on several spacecraft, including: REX-II, OAST-Flyer, GANE, Orbcomm, Microlab, and others.

The antenna separation distances are 0.61 m, 1.12 m, and 1.07 m, respectively. One antenna (in baseline 3) is located 0.23 m out of plane (below) the other three antennae. On the spacecraft, the

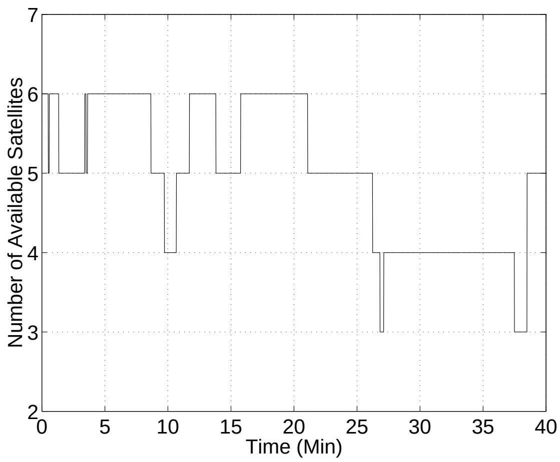

Figure 3: Available GPS Satellites

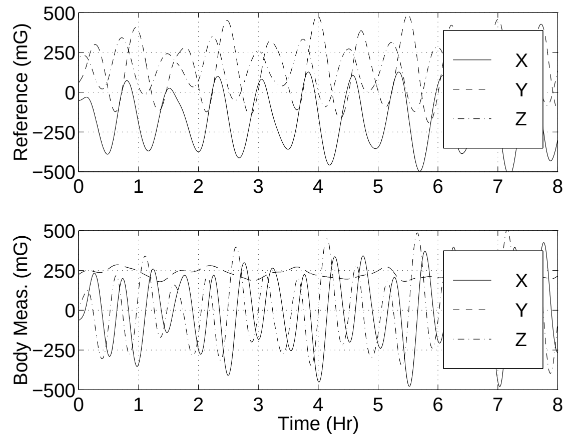

Figure 4: TAM Measurements

antennae are mounted on pedestals with ground planes to minimize signal reflections and multipath. For the simulation, multipath errors are introduced using a simple Markov process with a time constant of 5 minutes and a standard deviation of 0.026 wavelengths. The simulated spacecraft has four GPS antennae that form three baselines. The baseline vector components in wavelengths are given by

$$\mathbf{b}_1 = \begin{bmatrix} 2.75 \\ 1.64 \\ -0.12 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0.00 \\ 6.28 \\ -0.17 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} -3.93 \\ 3.93 \\ -1.23 \end{bmatrix} \tag{32}$$

Quantities such as line biases and integer ambiguities are first determined before the attitude determination algorithms are tested. The GPS raw measurements are processed at 1 Hz over a 40 minute simulation. A plot of the number of available GPS spacecraft for the 40 minute simulated run is shown in Figure 3. During the beginning of the run there are 5 to 6 available spacecraft. At the end of the simulation this drops down to about 4, which means that a degraded performance is possible (this also depends on the geometry of the GPS spacecraft, see Ref. [1] for Geometric Dilution of Precision).

The "true" magnetic field reference is modeled using a 10^{th} order International Geomagnetic Reference Field (IGRF) model.[19] Magnetometer measurements are known to be extremely accurate (within 0.3 mG). However, experience has shown that errors in the magnetic field are much larger (typically about an order of magnitude larger). Also, the errors are orbit dependent and nonlinear.[13] In order to simulate magnetic field modeling error, a 6^{th} order IGRF model with coefficients shifted by five years is used in the TAM measurement model. Also, TAM sensor noise is modeled by a Gaussian white-noise process with zero-mean and standard of 0.3 mG. A plot of the IGRF reference field and simulated TAM body measurements for an 8 hour run is shown in Figure 4. Typical roll-yaw coupling effects are shown in the X and Z axes.

The combined GPS/TAM integer ambiguity resolution approach has been implemented by evaluating the loss function in Eq. (18) every 20 seconds. Convergence is given when three times the square root of every diagonal element of Eq. (20) is less than 1/2. As mentioned previously only two baselines are required in this approach; baselines $\mathbf{b}_1$ and $\mathbf{b}_2$ are considered. The maximum possible integer for $\mathbf{b}_1$ is 3 and the maximum possible integer for $\mathbf{b}_2$ is 6. The actual integers are 1 for $\mathbf{b}_1$ and 3 for $\mathbf{b}_2$. Again, the search space has been significantly reduced since only two baselines are needed to determine the attitude with the combined GPS/TAM measurements. The loss function in Eq. (18) is evaluated for every integer pair, using the pseudo-phase measurement in Eq. (21) as the third measurement (with the "baseline" taken as the TAM body measurement). Convergence occurred after 200 seconds. Table 1 shows loss function values for possible integers pairs using the first available sightline over the 200 second time span (only the positive integer results are shown; the negative loss function values are all at least an order of a magnitude larger than the minimum

Table 1: First Sightline Loss Function for Candidate Integer Pairs

		$\mathbf{b}_1$			
		0	1	2	3
$\mathbf{b}_2$	0	3.78	1.66	5.63	17.20
	1	3.06	1.99	3.97	14.39
	2	1.35	1.40	3.03	12.58
	3	3.37	0.18	3.65	11.79
	4	5.09	2.75	5.04	12.00
	5	9.82	6.33	7.46	13.25
	6	15.56	10.91	12.89	15.52

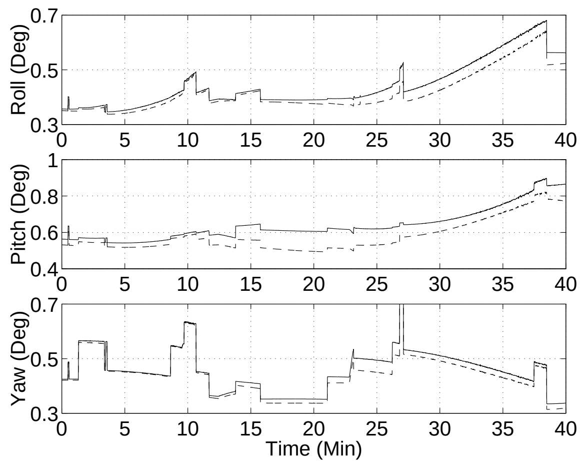

Figure 5: 3σ Outliers for GPS-only (solid line) and Combined TAM/GPS (dashed line)

value). Clearly, the minimum exists at the 1-3 integer pair, which has a loss function value of a least an order of magnitude smaller than any other value. This integer pair was then implemented as an initial guess in a nonlinear least-squares algorithm. With this initial guess the algorithm converges in 3 iterations to values of 1.9568 and 2.9481 for baselines $\mathbf{b}_1$ and $\mathbf{b}_2$, respectively. This indicates that the converged batch solutions are very close to the true integers. Also the 3σ outliers using Eq. (20) are given by 0.198, 0.255, and 1×10^{-5} for baselines $\mathbf{b}_1$, $\mathbf{b}_2$, and the TAM pseudo-baseline, respectively. Since all values are below 0.5 this indicates that rounding the least-squares found solutions to the nearest integers leads to the best possible solution. Also, the third outlier (1×10^{-5}) corresponds to the TAM bias. Since there is no TAM bias this value should be small, as indicated by the results. This again gives another integrity check for the approach. Similar results are also given for the second sightline. This approach has clear advantages over the motion-based technique in Ref. 11, which required over 6 minutes for the solution to converge.

Once the integers have been determined for the 2-baseline, 2-sightline case with TAM measurements then attitude determination is possible. Using these measurements the ALLEGRO algorithm, given by Eqs. (28)-(30), was implemented with an initial attitude guess given by the unity quaternion. We this initial guess the ALLEGRO algorithm converged in 5 seconds. Then the computed attitude was used to resolve the remaining integers associated with the other available sightlines. A study was then made which compared the solution using GPS-only measurements with the combined GPS/TAM measurements. Results for the 3σ attitude-error outliers for these two cases are shown in Figure 5. Clearly, using the combined GPS/TAM measurements gives better performance than GPS alone (especially in the pitch knowledge). Another advantage to using TAM measurements is that when combined with a dynamic model or gyros in a filter,[9] the TAM measurements can be used to determine 3-axis attitude during anomalous GPS situations, such as outages and re-initialization.

Conclusions

In this paper GPS measurements were combined with three-axis magnetometer measurements in order to quickly resolve the associated GPS integers for attitude determination. The algorithm only require two baselines in order to provide an attitude solution with TAM measurements. It uses an integer search approach to resolve the integers by evaluating an optimal loss function. The new algorithm has significant advantages over previously existing algorithms. First, the algorithm is attitude independent so that no a-priori attitude estimate (or assumed vehicle motion) is required. Second, since only two baselines are required the search space is significantly less than the case of using three baselines. Finally, several criteria can be used to check the integrity of the found solutions. Also, the TAM measurements can be used to update the attitude estimates using a filter, so that an attitude can still be determined when GPS measurements are not available. The algorithm was tested using a GPS hardware dynamic simulator to simulate the motion of a typical low-altitude Earth-orbiting spacecraft. Results indicate that the combined GPS/TAM approach provides a viable and robust means to effectively resolve the integer ambiguities and gives better attitude estimates than using GPS measurements alone.

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