Robust Aircraft Longitudinal Control Using Model-Error Control Synthesis

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In this paper a robust control for aircraft longitudinal motion is presented. The Model-Error Control Synthesis (MECS) is developed for this application, which consists of a nominal controller with a model-error predictive filter. The control input is updated directly using the estimated model-error from a predictive filter to cancel the unmodeled dynamics or disturbance inputs. The predictive filter is used only to estimate the model error, whereas an extended Kalman filter is used to provide state estimates from noisy measurements. The nominal control is chosen to be an LQR/dynamic inversion outer/inner loop controller for the longitudinal direction. The control law robustness to unmodeled dynamics and external disturbances is verified when MECS is active and not active. From various simulation results, MECS is shown to have better performance characteristics over the stand-alone nominal control.

Nomenclature

J	Performance index
f	Residual value vector
$\mathbf{x}$	Variable value vector
F_T	Thrust Force, N
m	Mass, kg
V	Airspeed, m/s
q	Angular rate, rad/s
θ	Pitch angel, rad
α	Angle of attack, rad
θ	Pitch angel, rad
δ_e	Elevator deflection, deg/s

Superscript

$\wedge$	Estimated value
$\sim$	Measured value

I. Introduction

The need for robust controllers has challenged researchers for different types of uncertainties and applications. Each uncertainty requires different types of robustness to be provided by the controller that may not be suitable for other uncertainties. For instance, robust control laws such as H_∞ and μ-synthesis controllers are designed to be less sensitive to external disturbances.[1,2] Meanwhile, adaptive control, neural network and fuzzy logic approaches are intellectual controllers.[3] Adaptive control allows for on-line parameter modification, which handles parameter variations. Fuzzy logic and neural network use learning adaptive rules. Model-error control synthesis (MECS) is another type of robust control.[4] It modifies the nominal control input rather than the model itself. MECS is designed to handle the uncertainty associated with

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external disturbances as well as unmodeled dynamics. It enhances the robustness of the nominal controller by feeding back information of the model error, which is estimated by a predictive filter.

In flight control, linear models are widely used for their computational advantage. Linear models limit the operation of the flight control by decreasing the flight envelope and the allowed maneuvers. Different models are used for different flight conditions. For robustness, researchers determined methods to update the linear model in real time. References 5 and 6 develop a rapid identification technique algorithm to find an optimal linear model for the required flight condition. An adaptive identification method to update a nonlinear model is presented in Ref. 7. Moreover, a study on longitudinal flight-control robustness in Ref. 8 is made between H_∞ and μ-synthesis control laws in the presence of weight and center of gravity uncertainty. This showed that both laws provide robust stability; but for some scenarios though, μ-synthesis guarantees the desired robust performance for the perturbed closed-loop system. Variable structure flight-control provides tracking control through local sliding surfaces.[9,10] This is an alternative to dynamic inversion techniques where no information of the model is needed.[10] Dynamic inversion (DI) is a straightforward method where the nonlinear dynamics are canceled and replaced with the desired stability dynamics that ensure cancelation of unmodeled effects. DI has been recently applied for many aircraft controllers.[11–14] Adams et al. designed a DI/μ-synthesis inner/outer loop control law for the thrust vectored F-18 in Refs. 15 and 16. In fact, it is very common for an aircraft controller to be designed to have an outer robust performance loop and an inner stability equalization loop. The inner loop, sometimes referred to as the stability augmented system (SAS), uses a simple feedback compensator, like proportional-integral-derivative (PID) control or DI control. On the other hand, the outer loop is used to achieve the desired performance and robustness characteristics.

Optimal estimation is very useful in engineering applications that involve noise or incomplete measurements. An estimator processes the measured data with a prediction model to achieve an optimal estimate of the truth. The most common real-time estimator is the Kalman filter.[17,18] It provides an unbiased estimate in the presence of noisy measurements and incomplete information. A nonlinear extension of Kalman filter is the extended Kalman filter (EKF).[19] The EKF assumes that the process noise is zero-mean white noise, which may not be realistic for many systems. Alternatively, a minimal model error (MME) algorithm deals with this incorrect assumption of the EKF process noise.[20] The MME approach determines the model error that is added to the model to have a more accurate representation of the truth without making any assumptions on the process noise. However, it is a batch filter, which cannot be executed in real time. In Ref. 4 a predictive estimator/filter is developed that essentially applies the concept of MME algorithm in a real-time manner using an approach introduced by Lu in Ref. 21. It optimizes for the solution of the cost function that consists of a weighted squared measurement-minus-residual and weighted squared model error. The predictive filter is applied to the spacecraft attitude estimation problem in Ref. 22.

MECS was first used as a robust control design for a nonlinear system in Ref. 23. The control system updates the nominal control input with the model-error estimate. A variable structure controller is chosen as the nominal controller and MECS is used to suppress the wing rock motion of a slender delta wing motion. Simulation results showed that MECS is very robust to external disturbances and uncertainties, and it can work with any nominal controller. MECS was successfully applied to the spacecraft attitude control problem in Ref. 24, using the non-adaptive portion of the controller from Ref. 25 with an EKF as a state estimator and a modified approximation recoding horizon (MARH) that determines the model error.

MECS showed promising results when it was used for robust control of spacecraft, where precise accuracy is required.[26,27] In this paper we apply MECS to the longitudinal flight-control problem, where larger model errors exists due to improper modeling and linearization, as well as larger external disturbances, are present than for spacecraft applications. MECS is used along with a nominal controller, which consists of a linearized feedback linear quadratic regulator (LQR) and DI in the inner loop. A nonlinear feedback linearized LQR is used as a nominal controller since it is known to be a very stable control law. However, it doesn't guarantee robustness. The main objective of this paper is to demonstrate the effectiveness of MECS on longitudinal flight-control robustness. Moreover, it illustrates a simple way to apply MECS to any existing control system.

The paper is organized as follows. First, the nonlinear feedback linearized LQR, as well as DI, are presented as the nominal controller for longitudinal flight. It is then followed by the mathematical principles of the MECS concept. Finally, the effects of applying MECS on aircraft longitudinal flight-control will be shown and discussed.

II. Aircraft Longitudinal Nominal Control

The control law shown here is a two-step controller. The outer loop is a LQR controller that tracks the commanded vertical position by generating a rate command and engine thrust. The inner loop is a DI controller associated with proportional-integral (PI) control that takes the rate command as an input and provides the proper control outputs. This kind of technique is one way of enhancing the control robustness by reducing the control law dependence on aerodynamics coefficients. The idea is to design separate controllers: one for the slow dynamics (outer loop) and another for the fast dynamics (inner loop). The aircraft nonlinear longitudinal equations of motion are given by the following form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} \tag{1}$$

where the state vector is $\mathbf{x} = [V, \alpha, q, \theta, x, y]^T$ and the input vector $\mathbf{u} = [F_T, q]^T$. The longitudinal nonlinear equation is written explicitly in the appendix.

A. Linear Quadratic Regulator (LQR) Control

The LQR controller is used for the outer-loop control of the longitudinal motion. We begin with the aircraft longitudinal equations and treat the angular rate q and the engine thrust F_T as input variables. Thus, by eliminating the equation for the angular rate from the equations of motion, we have a reduced order system of five states $\mathbf{x}_r = [V, \alpha, \theta, x, y]^T$. The reduced order is a representation of the slow dynamics. The dynamics equation for the reduced order system can be written as

$$\dot{\mathbf{x}}_r = \mathbf{f}_r(\mathbf{x}_r) + G_r(\mathbf{x})\mathbf{u}_r \tag{2}$$

where the input vector is $\mathbf{u}_r = [F_T, q]^T$. The reduced system is then locally linearized, since it is assumed that the aircraft flies in a near steady-state condition. Thus, the linearized system becomes

$$\Delta\dot{\mathbf{x}}_r = A\Delta(\mathbf{x}_r) + B\Delta\mathbf{u}_r \tag{3}$$

where $\Delta\dot{\mathbf{x}}_r$ represent the perturbation states, $\Delta\mathbf{u}_r$ is the input for the reduced system, A is the 5×5 system matrix, and B is the 5×2 input matrix. Next, the reduced system will be augmented by two states, which are the integrals of errors in position. By integrating the errors of position, perfect tracking is ensured in theory. As a result, the state vector for the augmented system becomes $\mathbf{x}_a = [V, \alpha, \theta, x, y, \int x - x_c, \int z - z_c]^T$. The augmented system is

$$\Delta\dot{\mathbf{x}}_a = A_a\Delta\mathbf{x}_a + B_a\Delta\mathbf{u}_r \tag{4}$$

Finally, the LQR controller is executed for the linearized augmented system. The performance index for the LQR designed for the augmented system is defined as follows:

$$J = \frac{1}{2}\int_{t_0}^{\infty} \Delta\mathbf{x}_a^T(t)Q\Delta\mathbf{x}_a(t) + \Delta\mathbf{u}_r^T(t)R\Delta\mathbf{u}_r(t)\ dt \tag{5}$$

where the LQR weighting matrices Q and R have been chosen after proper tuning to obtain the desired performance. The resulting stable tracking control law with zero steady-state errors and is now given as

$$\mathbf{u}_r = K\mathbf{x}_a(t) \tag{6}$$

where K is the optimal steady-state feedback gain matrix obtained from the LQR solution, with $K \equiv -R^{-1}B_a^T P$, where P is the solution to the associated steady-state Riccati equation.

B. Dynamic Inversion Control

Linear aircraft control design is challenging due to the nonlinear behaviors and uncertain aircraft dynamics. The conventional solution is to have a set of control design points at trim conditions, and then perform gain scheduling by interpolating gains with respect to aircraft conditions. Some nonlinear techniques have been explored to eliminate gain scheduling. For example, the DI method avoids the scheduling problem by using nonlinear feedback to cancel the dynamics of the aircraft. DI control has become popular in recent years for many aircraft applications. The DI approach cancels the undesirable dynamics and replaces it with a designed dynamic behavior. This is done by a proper feedback function; therefore, it is also called feedback linearization.

The inner loop controller uses DI augmented by PI control to compensate for the inversion error. It takes the angular rate command q as input and generates a control surface command to ensure tracking. The angular rate equation is

$$\dot{q}_r = f_D(\mathbf{x}_r, q) + g_D(\mathbf{x}_r, q)\delta_e \tag{7}$$

Thus, the angular rate equation is linear in terms of the angular rate q. The DI controller has to track a commanded angular rate q_c given by the LQR controller (outer loop). The integral part in the PI controller is used to guarantee a zero steady-state error. As a result, PI control provides the robust desirable behavior in the presence of disturbances. The error integral is defined as:

$$\dot{w} = q - q_c \tag{8}$$

Finally, the DI controller becomes

$$\delta e = g_D^{-1}\left(-f_D(\mathbf{x}_r, q) - K_{PI}\begin{bmatrix} q - q_c \\ w \end{bmatrix}\right) \tag{9}$$

where K_{PI} is a 2×2 gain matrix for the PI controller. The DI controller has some limitations. As with any linear technique, the number of independent input variables must equal or exceed the number of control outputs. Also, it cannot control non-minimum phase outputs. A zero on the right-hand plane will be inverted and appears as internal instability in the closed-loop system.

The overall nominal control consists of the outer-loop linearized LQR and DI as the inner-loop control. They are dynamically integrated together to have a uniform behavior at all operation conditions. The inner/outer-loop (DI/LQR) separately addresses operating envelope variation and robustness concerns. The outer loop contains a dynamic compensator that achieves the designed flying qualities, while the inner loop accounts for the system dynamics changes with flying conditions.

III. Model-Error Control Synthesis

The approach introduced here, called model-error control synthesis (MECS), provides robustness for nonlinear systems. MECS is not an explicit controller. Instead an existing controller is combined with an optimal estimator that determines a model-error correction. Next, the nominal control input will be updated with the estimated model error given by the predictive filter. In addition, the MECS technique does not need the system parameters to be updated to achieve robust performance required. Moreover, it can handle varying or unmodeled parameters as well as unmodeled disturbances.

A. Predictive Filter

MECS is based on a predictive filter that estimates the model error during the estimation process. Thus, MECS for aircraft combined with the nominal aircraft's controller provides robust performance. The predictive filter estimates for the unmodeled disturbance inputs. It provides an estimated input vector $\hat{\mathbf{d}}(t)$, called a model-error estimate. It can then be fed back and integrated forward in the equations of motion to give a better state estimate. It can also modify the control input to cancel the disturbance effects as well as the unmodeled dynamics. We follow a similar approach of Refs. 26 and 27 in the derivation of the predicted filter. Consider the following nonlinear system equation:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) + G(\mathbf{x})\hat{\mathbf{d}}(t) \tag{10}$$

$$\mathbf{y}(t) = \boldsymbol{c}\left(\mathbf{x}(t), \mathbf{u}(t)\right) \tag{11}$$

where $\hat{\mathbf{d}}(t)$ is the to-be-determined model error. Taking the Taylor series expansion for the output in Eq. (11) gives

$$\mathbf{y}(t + \Delta t) \approx \mathbf{y}(t) + \boldsymbol{z}(\mathbf{x}(t), \Delta t) + \Lambda(\Delta t)S(\mathbf{x}(t))\hat{\mathbf{d}}(t) \tag{12}$$

where the i^{th} row element of $\boldsymbol{z}(\mathbf{x}(t), \Delta t)$ is

$$\boldsymbol{z}(\mathbf{x}(t), \Delta t) = \sum_{k=1}^{p_i} \frac{\Delta t^k}{k!} L_f^k(c_i) \tag{13}$$

for $i = 1, 2, ..., m$, where m is the order of the lowest derivative of (c_i) where the control inputs first appears and $p_i = m$. The Lie derivative $L_f^k(c_i)$ is defined by

$$\begin{aligned} L_f^k(c_i) = &\quad c_i \quad &\text{for } k = 0 \\ L_f^k(c_i) = &\quad \tfrac{\partial L_f^{k-1}(c_i)}{\partial \mathbf{x}} \mathbf{f} \quad &\text{for } k \geq 1 \end{aligned} \tag{14}$$

The term $\Lambda(\Delta t)$ in Eq. (12) is a diagonal matrix for which the i^{th} element is given by

$$\lambda_{ii} = \frac{\Delta t^{p_i}}{p_i!}, \quad i = 1, 2, ..., m \tag{15}$$

and $S(\mathbf{x}(t))$ is the generalized sensitivity matrix for which the i^{th} element is given by

$$s_i = \left\{ L_{g1}[L_f^{pi-1}(c_i)], ..., L_{gq}[L_f^{pi-1}(c_i)] \right\} \quad i = 1, 2, ..., m \tag{16}$$

where the Lie derivative with respect to L_{gi} in Eq. (16) is defined by

$$L_{gj}[L_f^{p_{i-1}}(c_i)] = \tfrac{\partial L_f^{p_{i-1}}(c_i)}{\partial \mathbf{x}} g_j \quad j = 1, 2, ..., p \tag{17}$$

where p is the dimension of the vector input $\mathbf{u}(t)$ and g_j is the j^{th} column of the control matrix $G(t)$. Using the defined Lie derivatives with longitudinal equations of motion, the $\boldsymbol{z}$ vector is given by

$$\boldsymbol{z} = \begin{pmatrix} \Delta t \dot{V} \\ \Delta t \dot{\alpha} \\ \Delta t \dot{q} \\ \Delta t \dot{\theta} \\ \Delta t \dot{x} \quad + \frac{1}{2} \Delta t^2 (\frac{\partial \dot{x}}{\partial V} \dot{V} + \frac{\partial \dot{x}}{\partial \alpha} \dot{\alpha} + \frac{\partial \dot{x}}{\partial \theta} \dot{\theta}) \\ \Delta t \dot{z} + \quad \frac{1}{2} \Delta t^2 (\frac{\partial \dot{z}}{\partial V} \dot{V} + \frac{\partial \dot{z}}{\partial \alpha} \dot{\alpha} + \frac{\partial \dot{z}}{\partial \theta} \dot{\theta}) \end{pmatrix} \tag{18}$$

Also, the S matrix is expressed by

$$S = \begin{pmatrix} \mathbf{g}_V^T \\ \mathbf{g}_\alpha^T \\ \mathbf{g}_q^T \\ \mathbf{g}_q^T \\ \frac{\partial \dot{x}}{\partial V} \mathbf{g}_V^T + \frac{\partial \dot{x}}{\partial \alpha} \mathbf{g}_\alpha^T \\ \frac{\partial \dot{z}}{\partial V} \mathbf{g}_V^T + \frac{\partial \dot{z}}{\partial \alpha} \mathbf{g}_\alpha^T \end{pmatrix} \tag{19}$$

where

$$\begin{aligned} \mathbf{g}_V &= [L_{g1}[L_f^0(V)], L_{g2}[L_f^0(V)]]^T \\ \mathbf{g}_\alpha &= [L_{g1}[L_f^0(\alpha)],\ L_{g2}[L_f^0(\alpha)]]^T \\ \mathbf{g}_q &= [L_{g1}[L_f^0(q)],\ L_{g2}[L_f^0(q)]]^T \end{aligned} \tag{20}$$

The predictive filter is mainly used to find an optimal estimate for the model error. An optimal estimate will be found by minimizing a cost function of the difference between the predicted system output and the measured system output. Accordingly, the cost function consists of the squared desired-minus-actual residual and the square model-error estimate, and it will be minimized with respect to the model-error estimate. With the preliminary mathematics introduced we can derive the predictive filter. First, a cost function is defined as

$$J(\hat{\mathbf{d}}) = \frac{1}{2} \left[\tilde{\mathbf{y}}(t + \Delta t) - \hat{\mathbf{y}}(t + \Delta t)\right]^T R \left[\tilde{\mathbf{y}}(t + \Delta t) - \hat{\mathbf{y}}(t + \Delta t)\right] + \frac{1}{2} \hat{\mathbf{d}}^T W \hat{\mathbf{d}} \tag{21}$$

where $\hat{\mathbf{y}}$ refers to the estimated value and $\tilde{\mathbf{y}}$ to the measured value, and R and W are (assumed) diagonal weighting matrices.

The partial derivative of Eq. (21) with respect to $\hat{\mathbf{d}}$ is obtained by taking into account the following matrices identities

$$\begin{gathered}\frac{d}{d\mathbf{x}}\left(\mathbf{x}^T W \mathbf{x}\right) \equiv W\mathbf{x} \\ \frac{d}{d\mathbf{x}}\left[\frac{1}{2}(A\mathbf{x}+\mathbf{b})^T W (A\mathbf{x}+\mathbf{b})\right] \equiv A^T W (A\mathbf{x}+\mathbf{b})\end{gathered} \tag{22}$$

Thus, minimization of Eq. (21) requires the following equality condition:

$$(\Lambda(\Delta t)S)^T R\left(\boldsymbol{z}+\mathbf{y}(t)-\hat{\mathbf{y}}(t+\Delta t)\right)+(\Lambda(\Delta t)S)^T R\Lambda(\Delta t)S\hat{\mathbf{d}}+W\hat{\mathbf{d}}=0 \tag{23}$$

Solving for $\hat{\mathbf{d}}$ yields the optimal model-error estimate

$$\hat{\mathbf{d}}=[(\Lambda(\Delta t)S)^T R\Lambda(\Delta t)S+W]^{-1}(\Lambda(\Delta t)S)^T R\left(\boldsymbol{z}+\mathbf{y}(t)-\hat{\mathbf{y}}(t+\Delta t)\right) \tag{24}$$

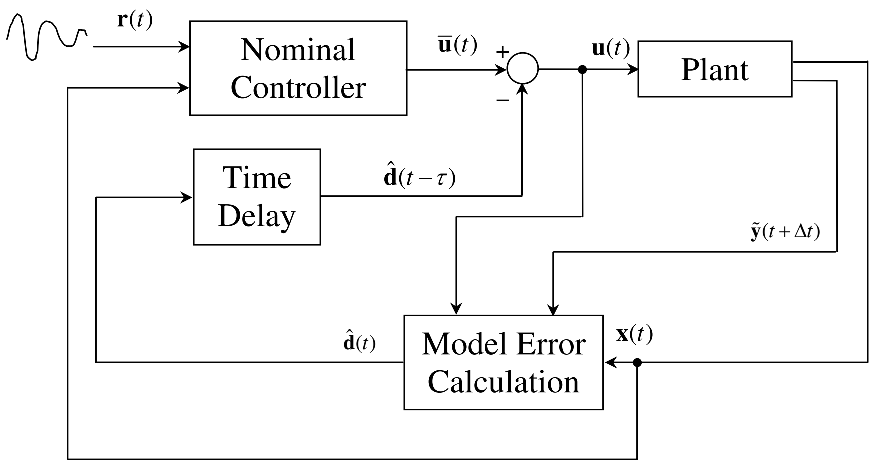


Figure 1. MECS Block Diagram

B. MECS Development

Consider the dynamics of the system to be modeled by

$$\dot{\mathbf{x}}(t)=\hat{\mathbf{f}}(\mathbf{x}(t))+\hat{G}(\mathbf{x})\overline{\mathbf{u}}(t) \tag{25}$$

However, the real system will be presented by

$$\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t))+G(\mathbf{x})\mathbf{u}(t)+E\mathbf{d}(t) \tag{26}$$

where $\mathbf{d}(t)$ is the model-error vector which is added from the real dynamic behavior of system to the equations of motion.

The main idea of the MECS approach is to feed back the estimated model-error vector to update the control input. The real model equation in Eq. (26) will be modified to be

$$\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t))+G(\mathbf{x})(\overline{\mathbf{u}}(t)-\hat{\mathbf{d}}(t-\Delta t))+E\mathbf{d}(t) \tag{27}$$

where $\overline{\mathbf{u}}(t)$ is the nominal control input vector, given by the LQR/dynamic inversion outer/inner loop controller, and $\hat{\mathbf{d}}(t-\Delta t)$ is the estimated model-error by predictive filter. MECS will cancel the unmodeled error as $\hat{\mathbf{d}}(t - \Delta t)$ becomes closer to $\mathbf{d}(t)$. Now, the dynamics of the true model in Eq. (27) will approach the dynamics of the estimated model in Eq. (25) and the overall control input will be in the form given by

$$\mathbf{u}(t) = \overline{\mathbf{u}}(t) - \hat{\mathbf{d}}(t - \Delta t) \tag{28}$$

The model error is taken at time $t - \Delta t$ because a response from the plant must be given before the model error can be determined. A schematic of the MECS approach is shown in Figure 1, where $\mathbf{r}(t)$ is a reference input.

A summary of the overall control design is now given. The MECS approach uses the predictive filter to estimate the model error, given by minimizing a cost function consisting of the weighted sum square of the measurement-minus-estimate residual and the weighted squared model-error estimate. Next, the estimated model error by the predictive filter is used to supplement the nominal control input. It is important to note that the MECS approach does not change the structure of the nominal controller or system parameters, it just modifies the nominal control input. The updated control input improves the ability of the true system to perform as designed.

IV. Simulation and Results

The aircraft used for the simulation is the F-18 HARV. The aircraft specifications and aerodynamics parameters are obtained from Ref. 28. This data is used in the nonlinear model developed earlier. The aircraft specifications are provided in Table 1 and aircraft aerodynamic parameters in Table 2.

Table 1. F-18 HARV Specifications

Parameters	Values	Units
Mass (m)	$14,792$	kg
Reference wing area (S)	37.98	m^2
Reference mean aerodynamic chord (MAC)	6.7	m
Reference span (b)	11.4	m
Pitch inertia (I_{yy})	15.33×10^6	N−m
Acceleration of gravity (g)	9.81	$\mathrm{m/s}^2$
Air density (ρ)	0.65360	$\mathrm{kg/m}^3$

Table 2. F-18 HARV Aerodynamics Parameters

Parameters	Definitions	Values	Units
C_{D_0}		0.0864	
C_{D_α}	$\partial C_D/\partial\alpha$	0.00310	1/rad
C_{L_0}		0.02	
C_{L_α}	$\partial C_L/\partial\alpha$	0.1	1/rad
$C_{L_{\delta e}}$	$\partial C_L/\partial\delta e$	0.01	1/rad
C_{M_0}		0.02	
C_{M_α}	$\partial C_M/\partial\alpha$	-0.0350	1/rad
$C_{M_{\delta e}}$	$\partial C_M/\partial\delta e$	-0.05944	1/rad
C_{M_q}	$\partial C_M/\partial(q\bar{c}/2V)$	-5.50	1/rad

During the simulation the aircraft performs an altitude change maneuver from 6000 m to 6600 m and then maintains a straight level flight. We assume all state measurements are available for all time. The aircraft initial state is $\mathbf{x}_0 = [V_0, \alpha_0, q_0, \theta_0, x_0, z_0]^T = [405, 0, 0, 0, 0, 6000]^T$. The aircraft maintains straight and level flight with a velocity of 405 m/s while the other states should be stabilized to their trim values. In the simulation, it is assumed that the measurement errors are represented by white Gaussian noise. Typical noise measurement values for each state are listed in Table 3. The noisy measurements are passed through

Table 3. Measurement Noise Parameters

State	Noise Standards deviation	Units
V	2	m/s
α	0.3	deg
θ	0.3	deg
q	0.003	rad/s
x	10	m
y	5	m

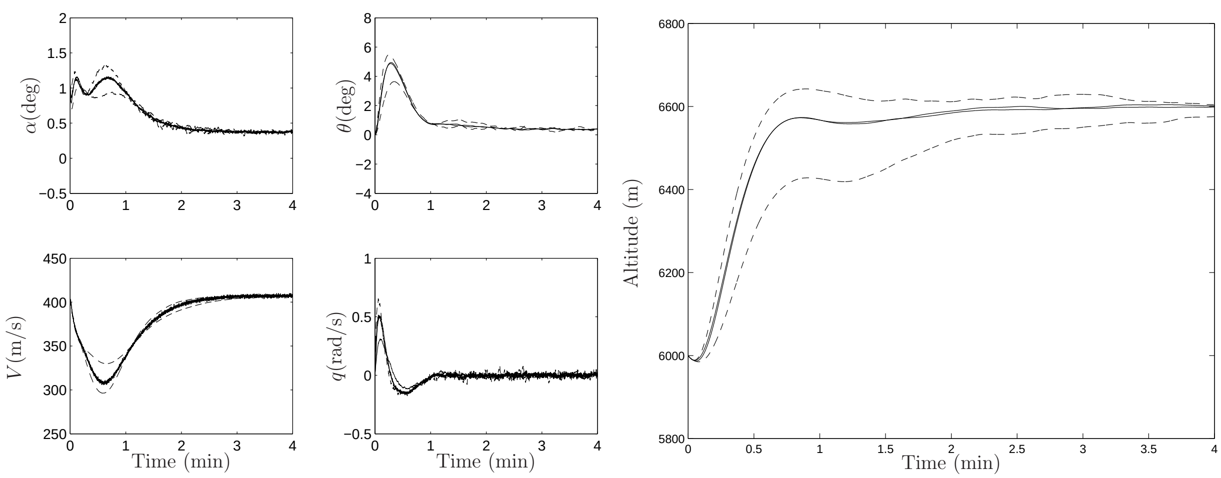
an EKF, which uses a stochastic process to provide estimated state values to be used by the nominal control as well as MECS.

The nominal control provides the necessary inputs for the maneuver. The LQR controller for the outer loop determines the optimal angular rate and engine thrust input. The LQR weighting matrices are chosen to be $Q = \text{diag}[1 \times 10^3, 0, 0, 10, 5 \times 10^3, 100, 5 \times 10^9]$ and $R = \text{diag}[1.5 \times 10^9, 1.5 \times 10^{20}]$. Note, a large weight is given for the angular rate input to limit the aircraft from performing aggressive maneuvers. Similarly, the inner-loop control will track the commanded angular rate q_c. The gains for the PI controller are $kp = -0.4$ and $ki = -0.0005$. At the same time, the predictive filter estimates the model error or external disturbance. MECS then uses the estimated model error to update the nominal control inputs at the next time step.

Several simulations using the longitudinal nonlinear model are performed. In the MECS approach the estimated model error is fed back to update the control inputs, thus mitigating the effects of uncertainties and external disturbances. Different types of uncertainties are implemented in the simulations to evaluate the MECS approach. The result when MECS is active is compared to when it is non-active (i.e. with nominal control only). An error function is used for evaluation purposes, where more weight is given at the end of simulation than at the beginning in order to gauge overall performance. The integral-sum-error (ISE) function is defined as

$$ISE = \sum_{k=1}^{t_f} (\Delta z_k^2 t_k), \qquad k = 1, 2, 3, ..., t_f \tag{29}$$

where Δz_k is the position error on the z-axis.



(a) Aircraft States

(b) Aircraft Position

Figure 2. Aircraft States History for MECS Off (Dash Line) and Active (Solid Line) with $C_{m_{\delta e}} \pm 40\%$ Error

One of the motives to use MECS is to provide robustness for unmodeled dynamics. Figure 2 shows the responses with the presence of model errors in the aerodynamics derivatives $C_{m_{\delta e}}$ and $C_{L_{\delta e}}$ of $\pm$%40 of their original values. It is clear that MECS provides robustness by compensating for the model error and cancels its effects on the system to behave as designed. Tables 4 and 5 show the difference that MECS makes to

Table 4. Error Results with Model Error of $-40\%\, C_{m_{\delta e}}$

	ISE	ISE STD	S.S. Error	S.S. Error STD
MECS off	3.2173×10^{10}	2.4353×10^{8}	26.0479	5.1942
MECS active	6.9567×10^{9}	2.5953×10^{7}	0.6035	2.4644

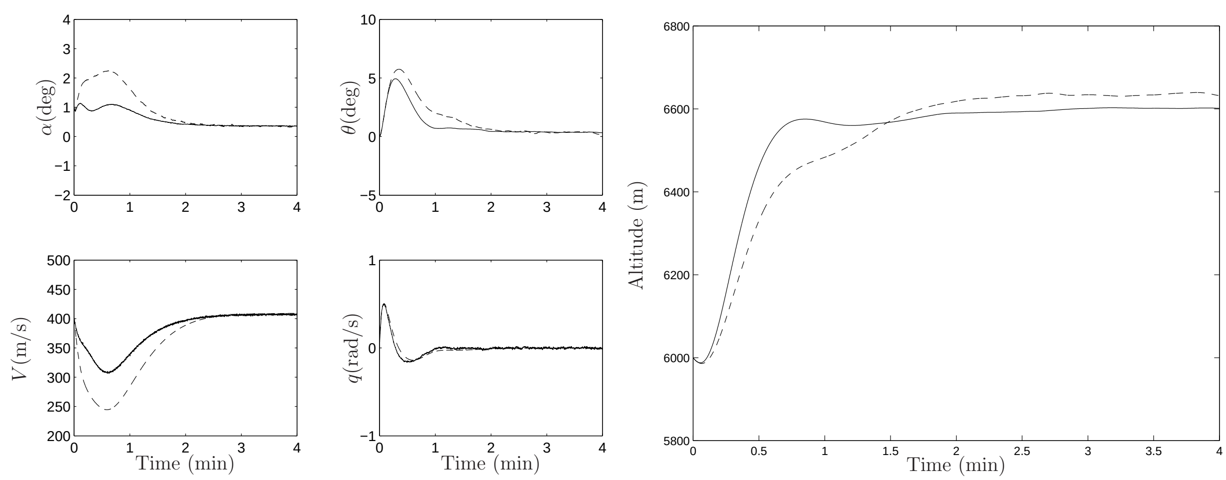
Table 5. Error Results with Model Error of $40\%,\ C_{m_{\delta e}}$

	ISE	ISE STD	S.S. Error	S.S. Error STD
MECS off	5.3487×10^{9}	9.0553×10^{7}	-9.0404	3.4704
MECS active	6.7377×10^{9}	2.8553×10^{7}	0.9783	2.8704

Table 6. Error Results with Model Error of I_{yy} +30% Model Error

	ISE	ISE STD	S.S. Error	S.S. Error STD
MECS off	6.937×10^{9}	5.5153×10^{7}	0.9521	1.9171
MECS active	6.8422×10^{9}	2.9813×10^{7}	0.7747	2.1341

the ISE and the steady-state error. MECS is also tested for another model-error case, where there is an error in the inertia matrix of 30%. From Table 6 it is seen that MECS does not offer much improvement for this case, as in previous case. This is because in the previous case the model error is more related to the control input than for last case and the nominal controller is able to handle this kind of uncertainty well. However, in both cases MECS provides improvements than using the nominal control alone. We should note that the LQR control law is well known to be very stable, but it lacks robustness, which can be increased by using MECS. Next, a different type of uncertainty is applied, which is an external disturbance of the wind component that acts in the opposite direction of aircraft's motion with the wind speed of 10 m/s. The results shown in Figure 3 and Table 7 confirm that MECS is also capable to make the necessary correction to the control law for external disturbances.



(a) Aircraft States (b) Position State

Figure 3. Aircraft States History for MECS Off (Dash Line) and Active (Solid Line) with 10 m/s Wind Gust

MECS clearly provides robustness for different types of uncertainties when no measurement noise is present. For the last scenario, we test the capability of the MECS approach when there is more than one type of uncertainty when the addition of measurement noise. Figure 4 shows the results of model errors ($C_{m_{\delta e}}$ and $C_{L_{\delta e}}$ of $-\%40$) and external disturbance (wind speed 10 (m/s)) plus measurements noise. Table 8 shows that the presence of measurements noise reduces MECS performance than in previous simulations;

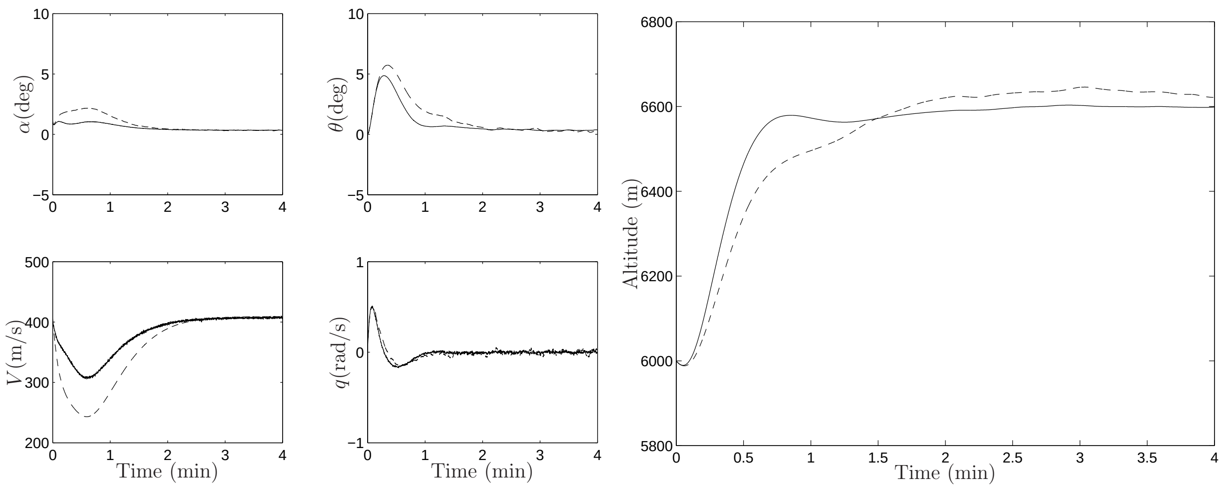
Table 7. Error Results with Wind Gust $W_x = -10$ (m/s)

	ISE	ISE STD	S.S. Error	S.S. Error STD
MECS off	1.6230×10^{10}	3.1611×10^{8}	-34.8642	4.2203
MECS active	6.7151×10^{9}	2.6039×10^{7}	0.4721	3.0515

however, the overall performance is much better than the nominal control alone. Two types of uncertainties have been applied, and MECS is able to provide good model-error estimation to mitigate their effects.

Table 8. Error Results with All Errors

	ISE	ISE STD	S.S. Error	S.S. Error STD
MECS off	5.5063×10^{10}	3.4611×10^{8}	-21.064	3.6203
MECS active	6.8207×10^{9}	2.4039×10^{7}	1.6672	3.1715



(a) Aircraft States (b) Position State

Figure 4. Aircraft States History for MECS Off (Dash Line) and Active (Solid Line) with All Errors

V. Conclusion

A robust control approach is introduced for the longitudinal aircraft motion, called model-error control synthesis (MECS). MECS is used to enhance the robustness of an excited nominal control and is not limited to the nominal control introduced in the paper. The model-error estimate by the predictive filter is used by MECS to update the nominal control input as a correction to a various errors in the system. For the model-error case, MECS showed a noticeable error reduction. This is due to the fact that the LQR controller provides stability, but is not robust to modeling errors. MECS provides significant improvements when external disturbances are present. On the other hand, the amplification of acceleration-level noise effects lowers MECS capability to cancel the model errors in the presence of measurement noise. However, MECS was still very effective when all types of uncertainty exist.

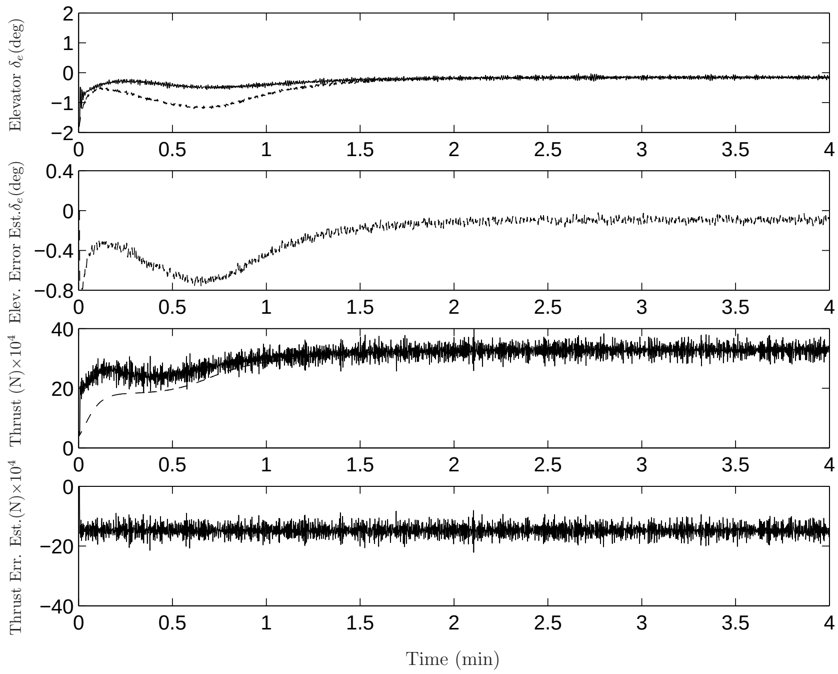


Figure 5. Aircraft Control Inputs and Predictive Filter Model-Error Estimates for MECS Off (Dash Line) and Active (Solid Line) with All Errors

Appendix

The aircraft longitudinal nonlinear equations of motion are given by

$$\begin{aligned}
\dot{V} &= \frac{1}{m}\left(-\overline{q}SC_D + F_T\cos\alpha\right) + g(-\cos\alpha\sin\theta + \sin\alpha\cos\theta) \\
\dot{\alpha} &= q + \frac{1}{mV}(-\overline{q}SC_L - F_T\sin\alpha) + g\sin\alpha\cos\theta \\
\dot{q} &= \frac{1}{I_{yy}}\left[\overline{q}S\overline{c}C_m + F_T\Delta Z\right] \\
\dot{\theta} &= q \\
\dot{x} &= V\left[\cos\alpha\cos\theta + \sin\alpha\sin\theta\right] \\
\dot{z} &= V\left[-\cos\alpha\sin\theta\ + \sin\alpha\cos\theta\right]
\end{aligned} \tag{30}$$

where C_D , C_L and C_m are the drag force, lift force and pitching moment coefficients, respectively, defined as follows

$$\begin{aligned}
C_D &= C_{D_0} + C_{D_\alpha}\alpha \\
C_L &= C_{L_0} + C_{L_\alpha}\alpha + C_{L_q}\frac{\bar{c}}{2V}q + C_{L_{\delta_e}}\delta_e \\
C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{b}{2V}q + C_{m_{\delta_e}}\delta_e
\end{aligned} \tag{31}$$

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