

# Space Collision Avoidance

October 2011

**Dr. John Crassidis and Dr. Puneet Singla**

Center for Multisource Information Fusion  
Department of Mechanical and Aerospace Engineering  
University at Buffalo  
Buffalo, New York

**Katie McConky and Dr. Moises Sudit**

Center for Multisource Information Fusion  
Department of Industrial and Systems Engineering  
CUBRC/University at Buffalo  
Buffalo, New York

## ABSTRACT

The U.S. Air Force collects information for the purpose of space surveillance through a global network of radars and optical sensors to maintain a catalog of over 20,000 resident space objects (RSOs), both active and inactive, with a minimum size of 10 cm. With the advent of better sensing technologies, such as the Space Surveillance Telescope and Space Based Space Surveillance satellite, much smaller RSOs can now be tracked than before. This will lead to the potential of at least an order of magnitude increase in the number of cataloged RSOs. A conjunction analysis is the process of quantifying the risk of close approaches between RSOs, which usually requires a probability of collision calculation. To perform this calculation the orbit states for every RSO with associated uncertainty must be maintained. Then comparative studies must be performed for two possible conjuncting RSOs. Several approaches, both heuristically- and statistically-based, have been developed to perform a conjunction analysis.

However, with the rapid increase in the number of cataloged objects, traditional conjunction analysis approaches to determine the collision potential between all the tracked RSOs will not be computationally tractable, due to the computational complexity required to complete a full analysis of all possible conjuncting RSOs. Simple pruning methods, such as not comparing RSOs at vastly different altitudes, will not reduce the computational burden sufficiently. This paper will detail a new parallelizable method that reduces significantly the number of comparisons required in order to effectively and quickly identify RSOs with potential collision paths. By projecting RSO paths into a hierarchical discretized space, objects requiring a more in-depth conjunction analysis are suggested based on their probability of being located in the same grid space at some future time. Computation reduction, system architecture and scalability will be discussed. Simulation results that validate the proposed methodology are shown.

## 1.0 Introduction

Space situational awareness (SSA) is a term used to describe awareness of both natural and manmade (possibly evasive) resident space objects (RSOs). Threats to the underlying assets must be suitably understood, detected and mitigated in order to develop plans for the “unknown unknowns” or, in the worst case, a “space Pearl Harbor” as stated by former Secretary of Defense Donald Rumsfeld. SSA involves both knowing RSO information, usually orbital position, and assessing how this information can affect future state information in relation to other RSOs. A common example involves assigning probabilities of collisions by an object with another one. An incident in Feb. 2009 involving an unintentional collision between Russia’s Cosmos 2251 satellite and a U.S. Iridium satellite underscores the need for SSA for RSOs. The satellite collision resulted in over 500 pieces of debris which pose an additional risk to remaining satellites. In Jan. 2007, China’s intentional destruction, using an anti-satellite test (ASAT), of one of its aging weather satellites set forth a new era of non-friendly SSA and threat assessment. China’s satellite destruction resulted in over 2,000 pieces of debris which spread quickly into many orbital regimes. Other collision examples can be found in Ref. [1]. Both natural and manmade RSOs pose serious threats to existing spacecraft assets, leaving our country’s critical assets significantly vulnerable.

Recently a report by the National Research Council (NRC) [2] underscores the importance of mitigating the adverse effects of space debris on operational satellites. The U.S. now tracks over 20,000 RSOs in space that are larger than 10 cm in diameter, all of which are in various orbits with velocities between 7 to 10 km/sec. Of these objects more than 900 are operational satellites. As of April 30, 2011 there are 470 satellites in low-Earth orbit (LEO), which constitute orbits that are typically less than 2,400 km altitude, 398 satellites in geosynchronous orbit (GEO), at roughly 36,000 km altitudes, 64 in mid-Earth orbit (MEO), at roughly 6000 to 20,000 km altitudes, and 34 in elliptical orbits.<sup>1</sup> The NRC report states that some scenarios indicate a tipping point for LEO objects, wherein all disposed space debris will collide with each other causing a cascade effect, which is known as Kessler’s Syndrome [3-4]. The problem is further compounded by the estimated 600,000 objects that are less than 10 cm and greater than 1 cm in size.

The Space Surveillance Network (SSN) is tasked with providing all the measurements needed for RSO tracking. It consists of an array of radio and optical telescopes distributed around the world. The SSN has a small number of dedicated sensors in the network, but also employs a large amount of the contributing components which were built for ballistic missile defense. The SSN data sources are provided mainly from ground-based radar and optical systems. Although these systems meet almost all tactical needs for cooperative Earth satellites, there are serious shortcomings for high-Earth orbit (HEO) and GEO objects [5]. Ground-based sensor networks are limited because large geographical gaps exist in the SSN coverage. Also, sensor detection sensitivity is low for HEO and GEO satellites and other distant space objects. This results in a slow response to intentional and unintentional non-cooperative maneuvers and breakup events, reducing SSA capabilities and the ability of gathering data for RSOs in higher orbits.

The Space Based Space Surveillance (SBSS) system will overcome many of these shortcomings by adding a space-based sensor in the SSN. The first SBSS component was launched Sept. 25, 2010 with a mean mission duration of 5.5 years (7 years design life). The SBSS system consists of a visible sensor payload using a large aperture with wide field of view. In addition it employs very low noise payload electronics. Since it is a space-based platform, which is not affected by weather, atmosphere or time of day, the SBSS will be able to see much dimmer objects than current ground-based platforms. Recent advances in optical technology have also been made for use in future ground-based platforms. For example, DARPA’s Space Surveillance Telescope (SST) employs a charge coupled device camera

---

<sup>1</sup> [http://www.ucsusa.org/nuclear\\_weapons\\_and\\_global\\_security/space\\_weapons/technical\\_issues/cs-satellite-database.html](http://www.ucsusa.org/nuclear_weapons_and_global_security/space_weapons/technical_issues/cs-satellite-database.html).

consisting of curved imagers and a high-speed shutter that allows for both fast scans and high sensitivity. This will offer a means to detect and track very small and very faint RSOs, providing space surveillance data in a matter of a few nights, rather than weeks or months that is currently required for existing ground-based telescopes. Taken together the SBSS and SST will provide significant advancements over existing technologies to improve SSA capabilities.

Spacecraft conjunction is defined as a close approach between two RSOs. The RSOs can be any type of object in space, including Earth-launched operating vehicles, debris or natural objects in orbit. A conjunction analysis combines observations/estimates of the position (and possibly velocity) at some time and uncertainties of these observations/estimates of the RSOs to propagate this information forward in time to the point of closest approach between two or more objects. Priority ranking are usually defined to assess possible actions to be taken for conjunctions between objects, such as a collision avoidance maneuver. However, spacecraft maneuvers are expensive in terms of fuel expenditures, which can decrease the lifetime of operating spacecraft. Thus, an accurate analysis is imperative to the overall conjunction process. But, a tradeoff usually exists between obtaining an accurate analysis and the required computational effort to perform this analysis.

This tradeoff will be an important topic in the near future because it is believed that the SBSS and SST will be able to track objects down to the 3-5 cm size range. It is also believed that the number of tracked objects will then increase by an order of magnitude over currently tracked objects. Several approaches exist to perform a conjunction analysis. For example, Refs. [6-8] present several algorithms based on Gaussian statistical measures. Reference [9] develops an approach when the relative motion is nonlinear in nature by using a contour integration methodology. The advantages and disadvantages of these and other approaches are discussed in Ref. [1]. Theoretically, for  $n$  RSOs  $n^2$  combinations must be evaluated to assess the possible conjunction between any two RSOs. Thus, computational burdens can be prohibitive to provide a completely automatic and real-time conjunction analysis. Reference [10] presents a number of approximations to analytically and/or efficiently compute the probability of collision between two RSOs, which produces reasonable upper bounds on the collision probability. However, this approach, much like other more computationally efficient approaches, is not appropriate for operational scenarios or detailed design problems, as it could generate an excessive false alarm rate or overly conservative designs under some conditions.

As seen by the aforementioned references and studies, much effort has been placed in recent years to provide reasonably accurate conjunction analyses while attempting to reduce computational burdens. All of the simplified methodologies investigate approaches to reduce the computational requirements of only the actual probability calculations required in the conjunction analysis. Very few approaches exist to determine what RSOs actually need a conjunction analysis in the first place. A simple approach to reduce/thin the  $n^2$  combinations is to calculate the absolute value between the largest perigee and smallest apogee between two RSOs and then use a simple distance tolerance. For example, LEO and GEO objects have no chance for close approaches. Reference [11] develops web service architectures to deliver relevant information on conjunctions to appropriately thin out large data sets. The approach is, however, still based on a miss distance calculation and the accuracy depends on the threshold chosen.

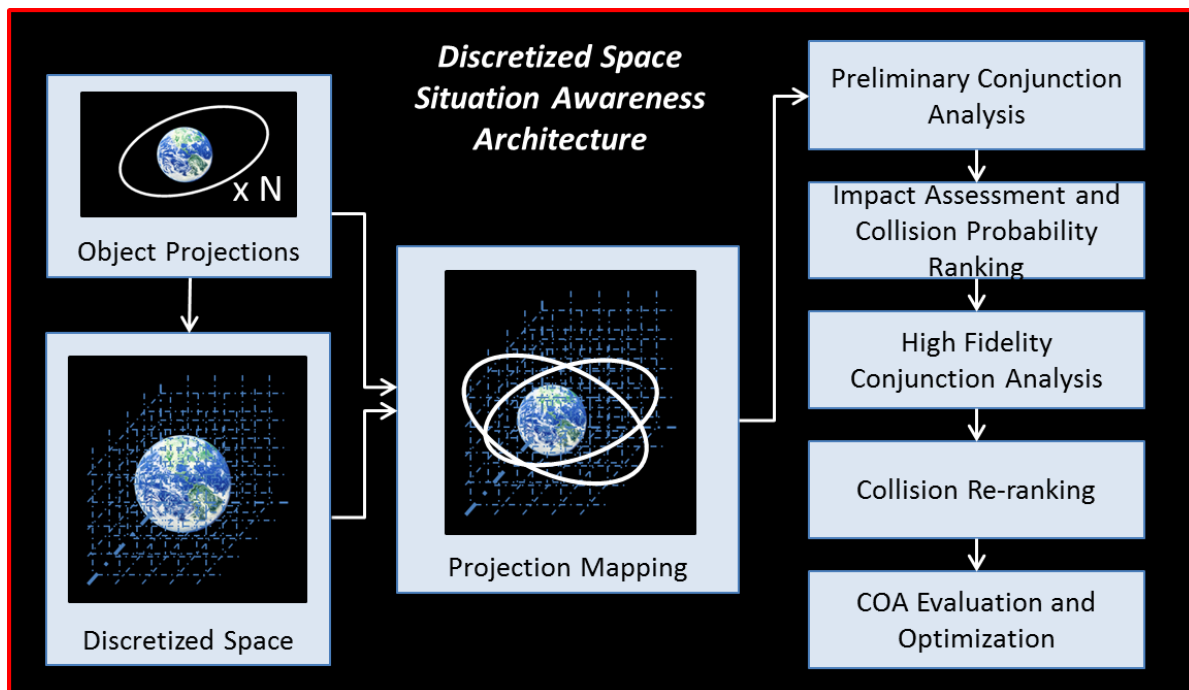
The main purpose of this paper is to provide a computationally efficient approach to reduce the number of combinatorial searches for cataloged RSOs. The approach provides computational efficiency through an algorithm that is easily parallelizable and automatically accounts for any orbit regimes, including LEO/GEO scenarios. It is important to note that the approach is independent of the specific chosen collision probability algorithm. The main purpose of the approach is to automatically find which objects should be chosen to perform a detailed conjunction analysis, which makes it useful as a general tool for any conjunction analysis algorithm. The heart of the approach lies in discretizing three-dimensional space and time, whose volume/box sizes are automatically set to achieve the desired granularity in terms of probability of collision accuracy in the present and future. An analytical closed-

form calculation is used to compute the probability that an object lies in a particular volume. Finding possible colliding objects is accomplished by comparing the probabilities for objects in the same volume. Furthermore, a higher-level data fusion engine is developed that can be used to assess whether or not two colliding RSOs are of relevance. For example, the Iridium/Cosmos collision would be automatically given a higher priority than the collision of two debris objects, assuming the debris collision will not impact any other assets.

The remainder of this paper begins by introducing the discretized space approach to conjunction analysis reduction in the framework of an overall space situation awareness architecture. The architecture description is followed by a discussion of the discretized space approach including a discussion of the parallelization of the process. Anticipated conjunction analysis reductions are discussed based on simulation results. We will also discuss the introduction of time-windows within the overall strategy of creating grids in space-time. Finally, the paper concludes with a discussion of the application of a high level data fusion engine coupled with mathematical modeling to prioritize and rank both conjunction analysis calculations and potential courses of action for satellite collision avoidance maneuvers.

## 2.0 Discretized Space Situation Awareness (DiSSA)

This section provides an overview of the entire space situation awareness architecture. The components of this architecture are meant to provide two core capabilities: an efficient means of performing a system wide conjunction analysis using the discretized space approach of DiSSA, and a data pipeline providing support for optimal course of action planning for mitigating potential collisions. A diagram of the architecture can be seen in Figure 1 **Error! Reference source not found.**, and each component is described in the remainder of this section.



**Figure 1: Discretized Space Situation Awareness Architecture**

An initial input to the system is a set of RSO orbit projections, with one path projection for each of the N space object under consideration. A single projection consists of a set of discrete points corresponding to the estimated path of an RSO. Each projection point should be minimally characterized

by a time stamp, a mean position, and an error ellipsoid, although non-Gaussian error distributions can also be handled. The determination of these RSO projections is out of the scope of this paper as many methods already exist and are being refined for forecasting the estimated paths of RSOs.

Once the system receives the RSO projections the first step is to discretize the space surrounding the earth by creating a series of boxes using a three dimensional grid. The earth is located in the center of this discretized space. The size of the grid spaces must be made with consideration to both the max speed of the RSOs and the window of time between projection points.

After receiving the RSO path projections and subsequently creating an appropriately sized grid space, each RSO projection is mapped to its corresponding set of boxes in the grid. A single projection point may map to several different boxes within the grid due to the uncertainty of the object's location. This process of mapping the RSOs' positions to grid boxes is highly parallelizable, and can be completed very quickly for RSOs with ellipsoid shaped error functions. This grid mapping process is the heart of the conjunction analysis reduction technique. Once all RSOs have been mapped to the discretized space, only those grid boxes that contain more than one RSO per time period are analyzed further. The main reduction in computation time comes from only needing to perform a conjunction analysis on those RSOs that end up in the same grid box during the same time period, rather than performing all  $N^2$  conjunction calculations.

Several methods for performing a conjunction analysis exist with some having more fidelity than others. To obtain a high fidelity conjunction analysis there is often a tradeoff in time due to the complexity of the mathematical calculations required. After the RSOs have been mapped to the discrete grid, a quick low fidelity conjunction analysis is done on all pairs of RSOs found inside the same grid space in the same time interval. Performing conjunction analyses on only those RSOs that align to the same grid space drastically reduces the number of conjunction analyses required, so that a full  $N^2$  set of conjunctions is no longer required, and the problem becomes much more tractable.

After the initial low fidelity conjunction analysis is done on all pairs of RSOs located in the same grid spaces, enough information is available to perform a preliminary collision ranking. RSO pairs are ranked in two separate manners, first by their probability of collision, and secondly by their potential impact if a collision were to occur. Using these two rankings and a modified Dempster-Shafer combination method, an ordered list of RSO pairs is created that ranks the pairs according to a combination of probability of collision and impact of collision.

Once the RSO pairs have gone under an initial ranking, the time remaining for calculation is used to further refine this list by re-estimating the probability of collision on the highest ranked RSO pairs using a high fidelity conjunction analysis technique. Since there may not be enough time to re-evaluate all RSO pairs with this high fidelity conjunction analysis, the initial probability and collision impact ranking helps to guide the system to the most important pairs to re-evaluate first.

Once the RSO pairs have been re-evaluated all pairs are re-ranked, once again according to the probability of collision and potential impact of collision. These two rankings mechanisms provide a decision maker with the ability to understand which pairs of RSOs are most likely to collide as well as which of the possible collisions could have the most impact if they actually were to collide. The impact of two RSOs colliding is related not only to the potential destruction of the two RSOs involved in the collision, but also to the impact of the resultant debris on nearby assets, and the relative importance of those affected assets. For example, the collision between two decommissioned satellites may not have much direct impact on the satellites themselves since they are no longer in use, but if the collision occurs in a manner such that debris from the collision could affect the orbit of a US military communications satellite the impact would become very significant.

The final aspect of the discretized space situation awareness architecture is a course of action analysis tool that provides an optimal course of action for avoiding potential collisions. This model looks to minimize the energy and financial costs of performing avoidance maneuvers while at the same time looking at the system as a whole and finding the minimum number of maneuvers necessary to avoid all collisions deemed significantly probable.

The remaining sections of this paper focus on the discretized space approach to conjunction analysis reduction as well as on methods to rank RSO pairs for further high fidelity conjunction analysis.

### 3.0 Space Discretization

The discretized space approach is used as a means to quickly and efficiently reduce the number of computationally expensive conjunction analyses required to analyze the collision probability of all objects within the system. The system is designed to quickly answer the question: “Of the  $N^2$  possible conjunction analyses I could do, which should I actually perform given the time I have available?”

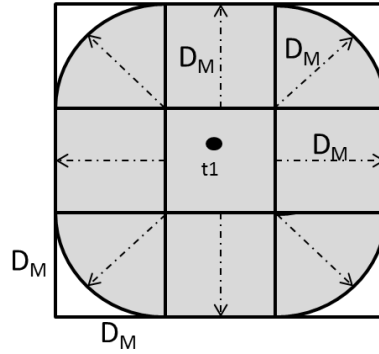
Some approaches look to reduce the number of conjunction analyses by performing simple filters, such as not comparing RSOs in GEO orbits with RSOs in LEO orbits. This research proposes using a similar approach, but instead of using only two boxes, a GEO box and a LEO box, the orbit space will be discretized into thousands of boxes. By gridding up the orbit space into small discrete boxes, each RSO can easily be mapped not to a single LEO or GEO box, but to a set of smaller boxes that correspond to the RSOs projected path. Grid boxes that contain more than one RSO present at the same time period provide a set of RSOs for which a conjunction analysis would be required. Instead of requiring  $N^2$  computationally expensive conjunction analyses, this system requires  $N$  simple mappings to the gridded space, and an additional set of conjunction analyses several orders of magnitude smaller than  $N^2$ . This section provides details on the projection process of discretizing the space as well as results based on a simulated dataset.

#### 3.1. Creating the Discretized Space

The discretized grid cannot be created arbitrarily, but rather must take into account the time interval between snapshots, and the maximum velocity of the RSOs being tracked. The minimum length of a grid space edge ( $D_M$ ) can be calculated by determining the maximum distance an RSO can travel between projection time stamps, as seen in equation 1, where  $v_{Max}$  is the maximum observed velocity among all RSOs, and  $\Delta T$  is the time interval between projection points.

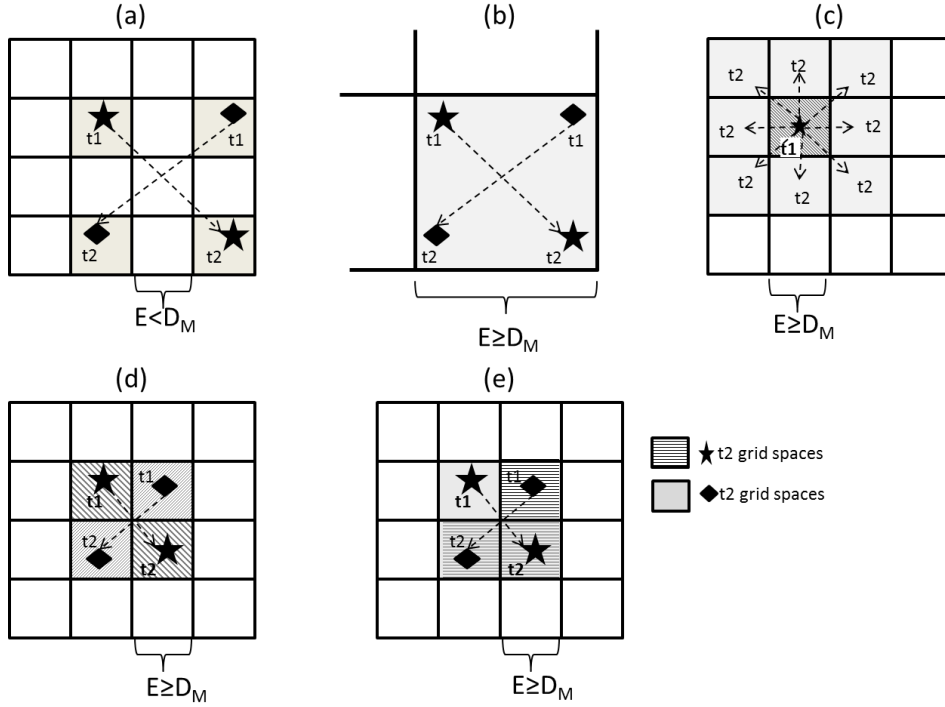
$$D_M = v_{Max} * \Delta T \quad \text{Equation (1)}$$

Forcing the discretized space to have a minimum edge dimension of  $D_M$ , allows the system to constrain the movement of an RSO between time stamps to only adjacent grid spaces. Figure 2 demonstrates in two dimensions the potential positions of an RSO at time  $t_2$  if the RSO were located somewhere in the center grid space during time  $t_1$ . Since  $D_M$  represents the most distance that an RSO could cover in the time between snapshots, it is impossible for the RSO to move further than the immediately adjacent grid spaces. Simple geometry supports that even moving in any direction from one of the grid space corners still keeps the RSO within the immediately surrounding grid spaces.



**Figure 2: Possible locations of RSO in 2D at time  $t_2$**

Figures 6a-d demonstrate in two dimensions some considerations and potential problems when designing the grid space. Figure 6a illustrates a potential problem that may occur if the grid size is set less than  $D_M$ . By setting the grid size to be less than  $D_M$ , Figure 6a shows that it is possible for RSOs to cross paths, and hence have a potential collision, without the mapping algorithm ever catching that the two RSOs may have possibly existed in the same grid space at the same time. Figure 6b demonstrates that if the RSOs pictured in 6a were instead mapped to an appropriately sized grid, such that the edge length was greater than or equal to  $D_M$ , then the two RSOs whose paths cross would be mapped to the same grid space, and thus would be candidates for a conjunction analysis. By making the minimum edge length of a grid be greater than or equal to  $D_M$ , the grid mapping ensures that an RSO can move no further than an adjacent grid space away within one time interval. The possible locations of an RSO at time  $t_2$  when the edge length of a grid space is set to be greater than or equal to  $D_M$  are depicted in Figure 6c as the shaded squares adjacent to  $t_1$ , and including the original  $t_1$  grid space. While restricting the minimum edge size to  $D_M$  ensures that the maximum position an RSO can move in one time interval is to an adjacent grid square, this does not entirely alleviate the problem of missed RSO path crossing events. Figure 6d depicts a potential instance where two RSOs cross paths, and hence have the potential for a collision, but the RSOs never share a grid space, and so therefore would not be recommended for a conjunction analysis. Figure 6e depicts an approach to remedy this potential problem by ‘completing the square’ when an RSO moves to a diagonally adjacent grid space between two time periods. The ‘complete the square’ approach automatically fills in the missing grid spaces in time  $t_2$ , so that there is one continuous shape between time  $t_1$  and  $t_2$ . If the ‘complete the square’ approach is done for both RSOs, as depicted in Figure 6e, both RSOs will be mapped to some of the same grid spaces in time  $t_2$ , and will therefore become candidates for conjunction analysis. When a third dimension is added the ‘complete the square’ approach extends to completing the cube if necessary.



**Figure 3: (a) Potential for missed collision if  $E < D_M$ , (b) Example of using a correctly sized grid for the figure (a) example, (c) possible locations of RSO at time  $t_2$ , (d) potential for missing collision if diagonal movement not accounted for, (e) the complete the square approach**

While there are penalties for making grid boxes too small, there are also penalties for making the boxes too large. Grid spaces where the edge size is significantly larger than the  $D_M$ , could result in too many conjunction analysis being recommended because the grid spaces will be so large that many RSOs that are actually very far apart could end up in the same grid space. Identifying the optimal size of the grid is a topic intended for future study.

### 3.2. Projection Mapping

The discretized space approach will only be useful if it takes significantly less time to map all the RSOs to the gridded space than it would to perform all  $N^2$  conjunction analyses. Fortunately, this is the case, even in the event of a large number of time intervals per projection. A closed form expression can be derived to determine the probability that an RSO will reside in a grid space. The probability of an object in some grid volume  $V$  can be computed by integrating the state pdf corresponding to that object over the grid volume  $V$ :

$$\Pr(x \in V) = \int_V p(x, t) dx$$

Integration of generic non-Gaussian pdf over a volume  $V$  is a computationally intensive process, however, if the state pdf is assumed to be Gaussian then one can compute this integral with a good accuracy. Genz et al. described many methods to compute multivariate probabilities over a hyper-rectangular integration regions [12]. Various methods based on acceptance-rejection, spherical-radial



transformations and separation-of-variables transformations are considered and it is concluded that a transformation developed by Genz [13] is the most efficient numerical method for computing multivariate normal probabilities for problems with as many as twenty variables. The MATLAB function mvncdf, which implement Genz's method, may be useful in evaluating the probability calculation. This function is used to quickly map the RSOs to the discretized space.

### 3.3. Experimental Results on Space Discretization

This section consists of two sections. Provided first is a description of the simulated dataset used to verify the space discretization approach. The simulated dataset was formed using 2000 RSOs from the Chinese anti-satellite missile test. The second section discusses the results of space discretization on conjunction analysis reduction using the simulated dataset.

#### 3.3.1 Dataset Simulation

The gridded space technology looks very promising for reducing the number of conjunction analyses necessary. To validate the algorithm a simulated dataset was provided using 2,000 RSOs from the debris of the 2007 Chinese anti-satellite missile test. These 2,000 RSOs were propagated forward in time using a two line element (TLE) [14] type simulation.<sup>2</sup> TLE state variables include mean elements of the eccentricity, inclination, right ascension of ascending node, argument of perigee, mean anomaly, and anomalistic mean motion. It also contains a terms that is analogous to a ballistic coefficient, known as  $B^*$ , which is given by [1]

$$B^* = \frac{1}{2} \frac{C_D S}{m} \rho_0 R_{\oplus} \quad \text{Equation (2)}$$

where  $C_D$  is the drag coefficient,  $S$  is the cross-sectional area,  $m$  is the mass,  $\rho_0$  is the atmospheric density at perigee of the orbit (assumed to be  $2.461 \times 10^{-5}$  kg/m<sup>2</sup>/ER), and  $R_{\oplus}$  is the Earth radius (ER), typically given by 6378.135 km. The ballistic coefficient,  $B$ , is defined by  $m/(C_D S)$  and is related to  $B^*$  by

$$B = \frac{R_{\oplus} \rho_0}{2B^*} \quad \text{Equation (3)}$$

Using  $R_{\oplus} = 6378.135$  km the constant conversion is given by

$$B = \frac{1}{12.741621B^*} \frac{\text{kg}}{\text{m}^2} \quad \text{Equation(4)}$$

NORAD SGP4 has a function that converts the TLEs to the osculating position and velocity of the space objects in the orbit with time.<sup>3</sup>

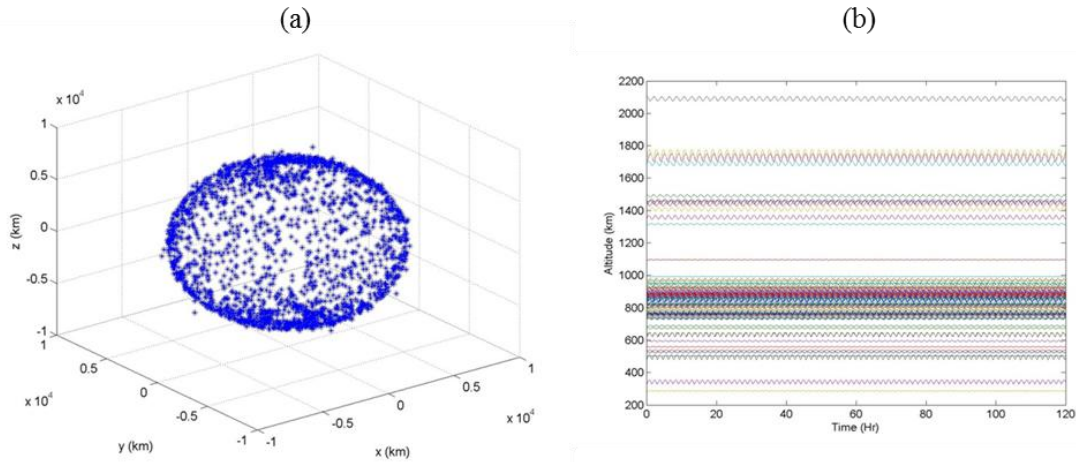
Each of the 2,000 RSOs is propagated for 5 days using an integration interval of 60 seconds. Note that TLE sets are usually updated daily but for the simulation purposes the original TLS is used throughout the 5 day propagation. A plot of the initial distribution of the RSOs at epoch is shown in Figure 4a. Figure 4a indicates that the breakup debris from the Chinese anti-satellite missile test is spread throughout the entire Earth's atmosphere. Figure 4b shows the distribution of the altitude for the 2000

<sup>2</sup> The format is described in [http://en.wikipedia.org/wiki/Two-line element set](http://en.wikipedia.org/wiki/Two-line_element_set).

<sup>3</sup> Code can be found at <http://www.zeptomoby.com/satellites/>.

RSO objects in the data set. Most of the RSOs are clustered between 800 km and 1000 km altitude, and thereby represent a dataset with many possible collisions.

The objects are propagated using an initial error process that involves a zero-mean Gaussian white-noise process with isotropic position standard deviation of 30 m and isotropic velocity standard deviation of 0.005 m/s. An unscented Kalman filter [15] is used to propagate the covariance forward in time, which in turn is used to compute the probability density function for the conjunction analysis.



**Figure 4: (a) Initial Distribution of RSOs, (b) Altitude Distribution for RSOs**

### 3.3.2 Results

This section provides details of the results of testing the discretized space approach on the simulated dataset. A simulation of the discretized space mapping algorithm was implemented in MatLab, and was used to validate the overall approach. While propagation was done at intervals of 60 seconds, focus was placed only at data corresponding to time snapshots of 1, 2, 3, 4, and 5 days out. To determine the size of the grid the maximum velocity of the RSOs and the time window between snapshots is required. The maximum velocity,  $v_{Max}$ , of all the RSOs was determined to be 7.8km/s. While the data was simulated in 60 second snapshots, a time snapshot of 10 seconds is more realistic for a live scenario, so a time snap shot of 10s was assumed. Using 7.8km/s and a time interval of 10s, the minimum grid size,  $D_M=78km$ . To simplify the mapping calculations, an edge size of 100km was used.

Results of processing the initial 2000 RSO dataset indicate that the number of conjunction analyses required can be reduced by over 4 orders of magnitude, over performing all  $N^2$  comparisons. To obtain this estimate we first looked at the number of boxes that each time snap shot mapped to, and chose to analyze further the day that mapped to the largest number of grid spaces. In the analyzed dataset, the snapshot for the start of day three mapped to the most boxes, with the 2000 RSOs mapping to a total of 6058 grid spaces. Using the day three time slice, random samples of data corresponding to 10 through 90 percent of the RSOs were taken, and the average number of conjunction analyses required for each data sample was calculated. This average number of conjunction analyses required per set of RSOs is seen in **Error! Reference source not found.** A polynomial curve provides the best fit for this data with an  $R^2$  value of greater than 0.99. In a worst case scenario it can be assumed that this pattern would continue as we added RSOs to the dataset. This can validly be assumed to be worst case because in reality not all RSOs would fall into the same altitude spectrum as the Chinese satellite debris, so this equation provides an upper bound on the number of required conjunction analyses.

Extrapolating the results of the 2000 Chinese satellite debris RSOs to 300,000 RSO objects the benefit of the discretized space approach for reducing the number of required conjunction analyses can clearly be seen. **Error! Reference source not found.** illustrates the dramatic difference between performing all  $N^2$  conjunction analyses versus using the discretized space approach to reduce the number of conjunctions required to only those between RSOs residing in the same grid spaces. While the discretized space algorithm still suffers from the same polynomial growth as a full conjunction analysis, the rate of growth is much lower due to the tiny coefficient residing in front of the squared term. When the number of RSOs is increased to 300,000 a full conjunction analysis would require 90 billion conjunctions, whereas the discretized space approach would require only 3.6 million, or only 0.004% of the possible conjunctions.

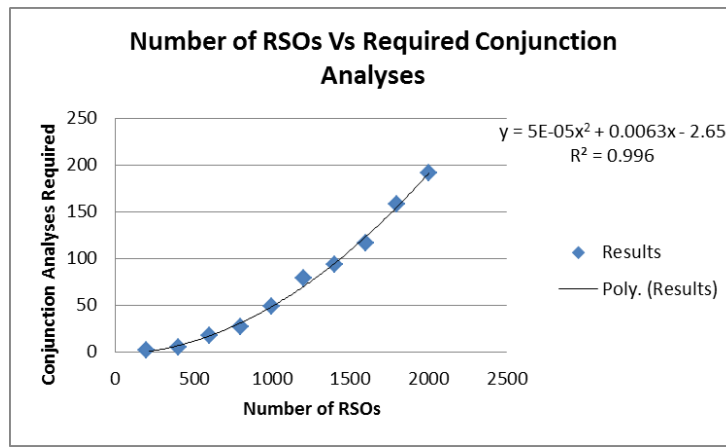


Figure 5: Number of Required Conjunction Analyses vs Number of RSOs

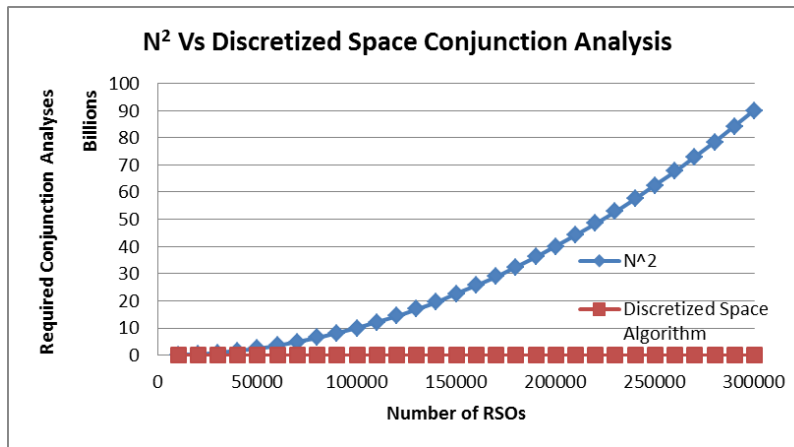


Figure 6: Discretized Space Conjunction Analysis Reduction Vs Full  $N^2$  Comparison

### 3.4. Parallelization of the Discretized Space Mapping

The discretized space approach lends itself nicely to parallelization during the mapping of projections to the gridded space. Firstly, each RSO can be mapped to the grid independently of the other RSOs, leading to the potential to use up to 300,000 processing threads, one for each RSO projection. As discussed previously, in order to avoid missing RSOs' that cross paths, mapping time  $T+1$  to the grid space requires knowledge of the mapping for time  $T$ . This does not prevent further parallelization

according to time however. It is still possible to multi-thread the mapping of each individual RSO by splitting the time interval up into several pieces. For example, splitting the entire time range up into 4 regions would require only 3 mappings to be repeated, but would get the RSO mapped for the entire time range completed 4 times faster. Finally, the final avenue for parallelization is through the conjunction analysis process. Each conjunction analysis could be processed separately without consideration for the other conjunctions that need to be done.

With the potential for such massive parallelization an architecture should be used that naturally lends itself to a highly parallelized environment. One such framework is the use of Hadoop map reduce processes residing on an HBase database. This framework is designed for dealing with volumes of large data, and is built to distribute massive data calculation requirements over a set of commodity machines, thereby allowing the computational power of a super computer without the capital investment required of a super computer. Additionally, since the mapping process is not necessary to do all the time, a cloud architecture may be applicable where space is paid for by minutes of use. Machines could be scaled up when a new set of projections comes in, in order to perform a very quick mapping, but then scaled back down while waiting for the next projection update. The exact implementation of the discretized space architecture is not the focus of this paper, although it is worth noting that systems exist today and are being further enhanced and developed to handle the amount of parallelization available in this framework.

#### 4.0 RSO Pair Rankings and Impact Assessment

While the discretized space approach provides a significant reduction in the number of conjunctions required, the estimated 3.6 million conjunctions for 300,000 RSOs for a single time snapshot is still a very large number of conjunctions for any system to handle. Realizing that there may not be enough time to actually complete even the reduced number of conjunctions, a ranking mechanism will be used to rank the potential conjunctions. One such ranking mechanism is to rank the potential conjunctions on the basis of probability of residing in the same box in combination with the potential impact of the collision.

While the two ranking mechanisms are useful on their own when combined together they become more valuable. For example, it makes no sense to perform a conjunction analysis between two RSOs with a very high impact score if they have very little probability of actually colliding. Therefore a mechanism is needed to combine the two measurements such that a single ranking is produced. This ranking should naturally float to the top of the list those RSO pairs that have a high probability of residing in the same grid space as well as have a high negative impact if a collision were to occur. To accomplish this we propose using a modified Dempster-Shafer (DS) combination method. To use this method each separate analysis, probability of collision and impact, is considered as a separate ‘vote’ that indicates how sure this individual analysis is that this pair of RSOs requires a complete conjunction analysis. The modified DS approach provides several advantages over traditional Dempster-Shafer including the ability to factor in model reliability, normalizing the data by conflicting information as well as uninformative information, and providing an intuitive conflict score based on a range of 0 to 1.

A brief overview of the modified DS method is presented here followed by a few sample calculations. The modified DS method starts off with the standard DS frame of discernment, where in this particular scenario the models are voting on whether or not to perform a conjunction analysis between two points. The frame of discernment,  $\theta$ , is seen in equation 5, and consists of Perform Conjunction and Do Not Perform Conjunction. The power set,  $2^\theta$ , consists of the set of all subsets of  $\theta$ , and therefore consists of Perform Conjunction, Do Not Perform Conjunction, Null, and Either Perform Conjunction or Do Not Perform Conjunction, as summarized in equation 6.

$$\theta = \{Perform\ Conjunction, Do\ Not\ Perform\ Conjunction\} = \{PC, \neg PC\} \quad equation\ (5)$$

$$2^\theta = \{PC, \neg PC, Null, Either PC or \neg PC\} \quad \text{equation (6)}$$

Each model provides an assessment of how strongly they feel the factors they are looking at warrant a conjunction analysis. Each model need only provide an assessment on the mass of Perform Conjunction,  $m(PC)$ , and a reliability ( $rel$ ) estimate for the model. All other values for the power set can be obtained from these two numbers by using equations 7 through 10.

$$m(PC) = \text{provided by model} \quad \text{equation (7)}$$

$$m(\neg PC) = 1 - m(PC) \quad \text{equation (8)}$$

$$m(\text{either}) = 0 \quad \text{equation (9)}$$

$$m(\text{null}) = 0 \quad \text{equation (10)}$$

As can be seen from equations 7 through 10 without factoring in model reliability the mass of the null set and the either set are forced to zero. This is because the models are forced to assign mass to the perform conjunction option, and any mass not assigned to perform conjunction is automatically assigned to the do not perform conjunction option, since those are the only two real options the system has available to it. However, when reliability is taken into account, the decision of the models becomes slightly unclear as there is doubt introduced as to the certainty of the model's assessment, this uncertain mass then gets attributed to the either set as seen in equations 11 through 14.

$$m(\text{null})|rel = m(\text{null}) * rel = 0 \quad \text{equation (11)}$$

$$m(PC)|Rel = m(PC) * Rel \quad \text{equation(12)}$$

$$m(\neg PC)|Rel = m(\neg PC) * Rel \quad \text{equation(13)}$$

$$m(\text{either})|Rel = 1 - m(PC)|Rel - m(\neg PC)|Rel \quad \text{equation (14)}$$

Once the reliability augmented mass function values are obtained, it is possible to calculate the dempster shafer combination values using a quasi-associative rule of combination [16], and finally a normalized value that will be used for ranking. The final necessary modified DS equations can be seen in equations 15 through 17. **Error! Reference source not found.** provides a comparison of the combined DS ranking method versus using each ranking method individually. In **Error! Reference source not found.**, notice in particular how two high scores combine to create an even higher score, and how the reliability of the models impacts the overall score as can be seen in pairs 3 and 4.

$$m_\cap^n(A) = \sum_{A_1 \cap A_2 \cap \dots \cap A_n = A \neq \text{Either}} m_1(A_1)|rel_1 * m_2(A_2)|rel_2 \dots m_n(A_n)|rel_n \quad \text{equation (15)}$$

$$m_\cap^n(\text{Either}) = 1 - [m_\cap^n(PC) + m_\cap^n(\neg PC) + m_\cap^n(Null)] \quad \text{equation (16)}$$

$$\text{normalized rank} = \frac{1}{1 - (m_\cap^n(\text{Either}) + m_\cap^n(Null))} * m_\cap^n(PC) \quad \text{equation (17)}$$

This ranking method can be used for both the initial pass at identifying which conjunction analyses to perform, as well as during the second pass to decide which conjunction analyses to refine.

**Table 1: Sample Modified DS Rankings Combining Probability Within Grid and Collision Impact Scores**

Pair ID	M(PC) Probability Model (rel = 0.9)	M(PC) Impact Model (rel = 0.9)	Rank According to Probability Model	Rank According to Impact Model	Rank According to modified DS with Score [rank (score)]
1	0.1	0.9	5	1	5 (0.50)
2	0.4	0.8	4	2	4 (0.685)
3	0.8	0.7	3	3	1 (0.863)
4	0.9	0.3	2	4	2 (0.717)
5	0.95	0.2	1	5	3 (0.703)

## 5.0 Conclusions

As space sensing technology continues to improve, thereby increasing the number of RSOs capable of being tracked, it is quickly becoming an intractable problem to perform a full  $N^2$  conjunction analysis of all RSOs. This paper has provided a method for efficiently identifying only those RSOs that have a potential for collision as candidates for a full conjunction analysis. By mapping RSO projected paths to a gridded space, it has been demonstrated that this method has the potential to reduce necessary conjunctions by over 4 orders of magnitude, requiring less than 0.004% of all  $N^2$  conjunctions to be completed. Additionally, improvements to space situation awareness have been suggested with novel ranking mechanisms to rank RSO pairs by not only probability of collision, but also in combination with the impact of the collision. Finally, a mathematical modeling approach was suggested in order to optimize collision avoidance maneuvers and improve space situational awareness.

This paper leaves much future work available for exploration. Future work includes verifying results on a larger dataset, preferably using around 300,000 RSOs. Additionally, an investigation needs to be completed to identify the optimal size of the gridded spaces, and the tradeoff between grid space size and computational efficiency and algorithm performance. Identifying and implementing the fully parallelized system remains on the table as well. There is much research to be done to identify a method to determine the impact of a collision between two RSOs, including efficient methods to simulate the effect of debris from such a collision on other high value space assets. Finally, an in depth investigation and outlining of a mathematical model for collision avoidance course of action planning needs to be explored and developed to complete the gridded space situation awareness architecture.

## References

- [1] Vallado, D.A., *Fundamentals of Astrodynamics, Third Edition*, Microcosm Press, Hawthorne, CA, and Springer, New York, NY, Chapter 11.7, 2007.
- [2] Committee for the Assessment of NASA's Orbital Debris Programs, *Limiting Future Collision Risk to Spacecraft: An Assessment of NASA's Meteoroid and Orbital Debris Programs*, The National Academies Press, Washington, DC, ISBN 978-0-309-21974-7, 2010.
- [3] Kessler, D.J., and Cour-Palais, B.G., "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt," *Journal of Geophysical Research*, Vol. 83, No. A6, June, 1978, pp. 2637-2646.

- [4] Scheffter, J., "The Growing Peril of Space Debris," *Popular Science*, July, 1982, Vol. 221, No. 1, pp. 48-51.
- [5] *Report on Space Surveillance, Asteroids and Comets, and Space Debris, Volume 1: Space Surveillance*, United States Air Force Scientific Advisory Board, Washington DC, June 1997, Report Number SAB-TR-96-04.
- [6] Alfano, S., "Determining Satellite Close Approaches: Part II," *Journal of the Astronautical Sciences*, Vol. 42, No. 2, Jan.-March, 1994, pp. 143-152.
- [7] Alfano, S., "Relating Position Uncertainty to Maximum Conjunction Probability," *Journal of the Astronautical Sciences*, Vol. 53, No. 2, April-June, 2005, pp. 193-205.
- [8] Alfano, S., "Satellite Collision Probability Enhancements," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 3, May-June, 2006, pp. 588-592.
- [9] Patera, R.P., "Satellite Collision Probability for Nonlinear Relative Motion," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 5, Sept.-Oct., 2003, pp. 728-733.
- [10] Carpenter, J.R., "Conservative Analytical Collision Probability for Design of Orbital Formations," *Proceedings of the 2<sup>nd</sup> International Symposium on Formation Flying Missions and Technologies*, Center for Aerospace Information, NASA CP-2005-212781, 2004.
- [11] Hall, R., Alfano, S., and Ocampo, A., "Advances in Satellite Conjunction Analysis," *Advanced Maui Optical and Space Surveillance Technologies (AMOS) Conference*, Maui, HI, 2010.
- [12] Genz, A., and Bretz, F., "Comparison of Methods for the Computation of Multivariate t Probabilities," *Journal of Computational and Graphical Statistics*, Vol. 11, No. 4, 2002, pp. 950–971.
- [13] Genz, A., "Numerical Computation of Rectangular Bivariate and Trivariate Normal and t Probabilities," *Statistics and Computing*, Vol. 14, No. 3, 2004, pp. 251–260.
- [14] Hoots, F.R., and Roehrich, R.L., "Spacetrack Report No. 3: Models for Propagation of NORAD Element Sets," *Aerospace Defense Center, Peterson Air Force Base*, Dec. 1980.
- [15] Julier, S.J., Uhlmann, J.K., and Durrant-Whyte, H.F., "A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators," *IEEE Transactions on Automatic Control*, Vol. AC-45, No. 3, March 2000, pp. 477–482.
- [16] Florea, M. et al., "Robust Combination Rules for Evidence Theory," *Journal of Information Fusion*, Vol. 10, No. 2, April, 2009.