Probability Hypothesis Density Filter Based Design Concept: A Survey for Space Traffic Modeling and Control

Quang M. Lam[1]
Orbital Sciences Corporation
Dulles, VA 20166

John Crassidis[2]
University at Buffalo
Buffalo, NY

The Probability Hypothesis Density (PHD) filter has been recently received a lot of attention by the estimation and data fusion community for its ability to provide a useful solution to the Bayesian filter problem (i.e., implementation issue). Its core foundation to other parallel directions, such as the Sequential Monte Carlo PHD, the Gaussian Mixture PHD and others, offers a viable path to practically implement and realize this promising technology. Potential key solutions offered by a PHD based design paradigm include: (1) non-Gaussian noise and correlated noise process mitigation; (2) replacement of the Extended Kalman Filter (EKF) linearization process to improve numerical instability; and (3) substitution of current convoluted multiple-target multiple-sensors mainstream solutions (e.g., bookkeeping of EKFs on the track file side coupled with a large probability/hypothesis combinatorial computation) with a compact and computational efficient solution framework (i.e., *only need one PHD filter which processes a random finite set as a meta target estimate using one meta sensor consisting of multiple sensors being stacked in a "measurement set"*). This paper provides a survey on mainstream multiple-target multiple-sensor tracking algorithms and uses those for a baseline comparison to evaluate the potential payoff of the emerging PHD design framework. Potential benefits of a PHD based design framework *via the single meta target single meta sensor concept* and current emerging missions, such as near Earth object tracking, space traffic modeling and control, and space situational awareness, will also be discussed in the context of closely spaced objects and large target population beyond typical terrestrial domain applications.

I. Introduction

A. General Discussion of Multiple Target Multiple Sensor Detection and tracking

There are eight major well known design challenges associated with the closed-loop multiple target multiple sensor (MTMS) detection and tracking system. They are listed as follows:

> *(1)The nonlinear dynamic behavior of the object itself (i.e., state uncertainty due to process noise influencing input; target maneuvering uncertainty. Modeled by the process noise covariance matrix Q);*
>
> *(2) The measurement uncertainty due to sensor dynamic and its bandwidth capturing capacity (i.e., measurement noise uncertainty modeled by the covariance matrix R);*
>
> *(3) Filtering and Prediction Process itself (i.e., linear filtering vs nonlinear filtering techniques selected to address the first two design challenges)*

[1] Lam.Quang@Orbital.com, Ph. D.
[2] johnc@buffalo.edu, Ph. D.

(4) The front end measurement to object's state vector estimate association or fusion;

(5) The back end object's state vector estimate to state vector estimate fusion (i.e., track to track fusion)

(6) Closely Spaced Objects (CSOs) Detection and Tracking

(7) Target status uncertainty (i.e., the number of target variation; its appearance/disappearance in the sensor FOV or death/birth by its nature or control by operator)

(8) Target/Object Identity Identification (and it is quite a challenge for space traffic environment subject to (i) which object needs to have its identity identified and which object does not need and (ii) debris and unannounced foreign launch satellite/object)

The first three challenges are usually tied together and being addressed under the filtering design framework. For instance, Kalman filter based design technique and its implementation variants can be found in [1]-[4] which capture the basic essence of how those three can be designed to address the MTMS tracking system. It is worth pointing out here that tuning of the Q and R matrices in order to address efficient MTMS performance was actively investigated by many researchers [5]-[8] (also see Figure 6 for an illustration of how Q and R are being processed using a traditional EKF scheme). Today, how to tune the Q and R matrices in order to reach an optimal (or suboptimal) performance is still being viewed as an art rather than a mathematical science based algorithm. Challenges 4 and 5 (i.e., measurement to track fusion and track to track fusion) become even tougher to address when multiple distributed and disparate sensors are being taken into account and the number of targets (to be detected and tracked) exceeds the typical terrestrial tracking applications (i.e., larger than 5000). Challenges 6 (i.e., CSOs) and 7 (target death and birth) are the most challenging subject of all for the space traffic modeling and control problem and they are the crux of this paper's main interests to introduce the Random Finite Set (RFS) and Probability Hypothesis Density (PHD) filter to address those two design challenges. These two challenges will be discussed throughout the entire paper in the context of track initiation, track fusion/merging, and track deletion on the track file side of the MTMS tracking system to assist future space traffic controller/operator for their traffic management decision subject to traffic alert message generation and collision avoidance maneuvering request.

B. Challenges of Space Traffic Modeling and Management

Space situational awareness (SSA) is a term used to describe both friendly and non-friendly (possible evasive) space vehicles (SVs) and other objects, such as space debris. SSA involves both knowing SV and object information, usually orbital position, and assessing how this information can affect future state information in relation to other SVs. A common example involves assigning probabilities of collisions by an object or SV with another SV. A recent incident in February 2009 involving an unintentional collision between Russia's Cosmos 2251 satellite and a US Iridium satellite underscores the need for SSA for friendly SVs. This resulted in over 500 pieces of debris which pose an additional risk to satellites. China's intentional destruction of one of its aging weather satellites set forth a new era of non-friendly SSA and threat assessment (TA). Note that this collision left about 2,500 pieces of debris in Earth orbit.

With the advent of improved space sensing technologies such as the Space Surveillance Telescope and the Space Based Space Surveillance satellite, the catalog of tracked resident space objects (RSOs) has the potential to grow from 20,000 RSOs to over 300,000 RSOs. The collision between Iridium 33 and Cosmos 2251 communications satellites and the constant threat of micrometeoroid and orbital debris collision in low-Earth orbit to national and international assets underscores the need for accurate SSA, including support to identify potential RSOs collisions and to provide optimal course of action planning in order to mitigate such situations. The order of magnitude increase in tracked RSOs will prevent current approaches from performing accurate data association.

Extensions of data association methods used for air traffic control (ATC) for RSO tracking are arduous in nature due to the much larger number of objects to be detected, tracked, monitored, and managed in space versus jetliners. Furthermore, the technology for space tracking does not exist compared to ATC in the context of transponder communication between aircraft to aircraft and aircraft to air traffic control. Object identity is a challenge for both aircraft [19] and RSOs though. Aircraft trackers typically employ a Kalman filter based tracking system while space traffic detection and tracking still use least square batch processing with man in the loop sorting for data association. Up to 30% of RSOs are associated using subject matter experts. This is due to the fact that space tracking data is sparse [20] by nature make the object detection and tracking in real time a challenge.

C. Need of Advanced Nonlinear Filtering With Random Finite Set as Space Traffic Modeling and Management

The mainstream MTMS data association and fusion algorithms described in [1] to [4] also exhibit some performance limitations. These include but are not limited to (i) complex multiple steps data association at the front end (i.e., correct measurement to the right track association for track update in the context of multiple sensors multiple target tracking and fusion) and back end (i.e., track association and fusion in the context of track fusion architecture at the track file levels either implemented in the centralized or decentralized architectures in order to produce a global cohesive consistent track accuracy at the system level). Limitations per algorithm are summarized in Table 1. The key summary statement for all of them is that they are no longer suitable for future space traffic modeling and tracking due to the large Resident Space Object (RSO) population.

Table 1: Limitations of Mainstream Conventional Multiple Target Tracking Systems

Algorithm Name	Shortcomings & Comments
Global Nearest Neighbor Filter (GNNF) & Its Variants	(1) Non Probability Approach, (2) may overlook some feasible association cases (i.e., miss-association), (3) require a large number of EKFs to be processed on the track file side due to a large RSOs population, (4) robust association against False Alarm (FA) rate (beating MHT) but susceptible to low probability of detection. **Clearly not feasible for SSA application which has an RSO population of larger than 15,000 for normal size and much bigger than that when accounting for smaller size of RSOs (3cm to 5 cm diameter range)**
Joint Probability Data Association Filter (JPDAF) & Its Variants	(1) The number of targets is fixed and known ;(2) the complexity of the calculation for joint association probabilities grows exponentially with the number of targets and the number of Measurements; (3) unable to perform track initiation and maintenance. **The above three points make JDPA unsuitable for Space Traffic Modeling and Control mission due to large RSO population.**

Multiple Hypothesis Tracker (MHT) & Its Variants	(1) Deferred or built-in combinations of measurement to target associations; (2) An exhaustive association of all received measurements (past and present) to either a single track or as clutter is known as a hypothesis (and needed for all tracks and clutters!); (3) At each time step, the MHT filter attempts to maintain a small set of hypotheses with high posterior probability; (4) When a new set of measurements arrives, a new set of hypotheses is created from the existing hypotheses and their posterior probabilities are updated using Bayes rule; (5) Note that in the generation of new hypotheses, a measurement can be assigned either to clutter, an existing track or a completely new track; (6) able to handle unknown target, target birth/death, and varying number of targets; (7) In practice, traditional implementations of MHT usually require validation/gating of measurements as well as heuristic pruning/merging of hypotheses to reduce computational requirements; (8) hypothesis is not observable indeed and impossible to measure it . **The combinatorial nature of MHT is its biggest limitation since the total number of possible hypotheses increases exponentially with time, thus unsuitable for SSA application**
Multi-Target Bayes Filtering Algorithm	(1) involving the evaluation of multiple set-integrals and with a large amount of targets, it is impossible to carry out that task; (2) Intractable due to heavy computational loads; (3) PF-SMC based can solve the first two limitations but still remains as a big hurdle; (4) not even being considered for terrestrial applications. **Not a good candidate for Space Traffic Modeling and Control application**

II. Nonlinear Filtering Based Multiple Target Tracking Algorithm Survey

A. Extended Kalman Filter and Unscented Kalman Filter

The Extended Kalman Filter (EKF) applies the Kalman filter to nonlinear systems by linearizing the nonlinear models (i.e., nonlinear dynamic model f and nonlinear measurement h described next, [26]) so that the traditional linear Kalman filter equations can be applied.

$$\begin{aligned} \frac{d}{dt}\dot{\vec{X}}(t) &= f(t,\vec{X})\cdot\vec{X}(t) + L\cdot\vec{w}(t) \\ \vec{z}_m(t) &= h(\vec{X}) + \zeta(t) \end{aligned} \tag{1}$$

The nonlinear dynamic model f for a space based object is formulated as follow. The position and velocity vectors of the target relative to the sensor platform, i.e., $X(t)=[r_t(t)\ \ v_t(t)]^T$, expressed in an ECI frame are written as follows:

$$\begin{aligned} \frac{d}{dt}\vec{r}_t(t) &= \dot{\vec{r}}_t(t) = \vec{v}_t(t) \\ \frac{d}{dt}\vec{v}_t(t) &= \dot{\vec{v}}_t(t) = -\mu\frac{\vec{r}_t(t)}{r_t^3} + w(t) \end{aligned} \tag{2}$$

where $r_t(t)$ and $v_t(t)$ are target or RSO position and velocity, respectively and μ is the gravity constant and w(t) is the state process noise accounting for both gravitational and non-gravitational acceleration uncertainties.

Rewriting equation (2) in terms of state space continuous dynamic expression as follows,

$$\frac{d}{dt}\dot{\vec{X}}(t) = \begin{bmatrix}\dot{\vec{r}}_t \\ \dot{\vec{v}}_t\end{bmatrix} = \begin{bmatrix}0_{3x3} & I_{3x3} \\ f(r,t)) & 0_{3x3}\end{bmatrix}\begin{bmatrix}\vec{r}_t \\ \vec{v}_t\end{bmatrix} + \begin{bmatrix}0_{3x3} \\ I_{3x3}\end{bmatrix}\vec{w}(t) \tag{3}$$

The term f(r,t), expressed in equation (3), is a nonlinear function per axis and exists only for the diagonal terms. Equation (3) will be used as the dynamic predicted model for the EKF when we linearize the nonlinear function *f* about its current state vector as first order Taylor Series expansion. It will be carried out in the complete EKF formulation once we introduce the nonlinear measurement model as follows,

$$z_m(t) = \begin{bmatrix} \rho(t) \\ \beta(t) \\ el(t) \end{bmatrix} = h(\vec{X}(t)) + \zeta(t) = \begin{bmatrix} \mathbf{h_1}(\vec{\mathbf{X}}) \\ \mathbf{h_2}(\vec{\mathbf{X}}) \\ \mathbf{h_3}(\vec{\mathbf{X}}) \end{bmatrix} + \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \zeta_3(t) \end{bmatrix}$$

$$or \quad z_m(t) = \begin{bmatrix} \sqrt{\rho_s^2 + \rho_E^2 + \rho_Z^2} \\ \tan^{-1}\left(\dfrac{-\rho_s}{\rho_E}\right) \\ \sin^{-1}\left(\dfrac{\rho_z}{\rho}\right) \end{bmatrix} + \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \zeta_3(t) \end{bmatrix} \tag{4}$$

where $h_1()$ is the range ($\rho(t)$), $h_2()$ is the azimuth angle ($\beta(t)$), and $h_3()$ is the elevation angle ($el(t)$).

The EKF framework for a single object tracking is then accomplished using the processing flow described in Section 4 of this paper while multiple object measurements are being sorted out and associated with the right EKF using the Global Nearest Neighbor (GNN) data association scheme presented in Figures 5 & 6 of Section IV.

B. Multiple Model Based Estimator

Multiple-model adaptive estimation (MMAE) is a recursive estimator that uses a bank of filters that depend on some unknown parameters [21]. For example these parameters can be the elements of the process noise covariance, denoted by the vector **p**, which are assumed to be constant (at least throughout the interval of adaptation). Note that we do not necessarily need to make the stationary assumption for the state and/or output processes though, i.e. time varying state and output matrices can be used. A set of distributed elements is generated from some known pdf of **p**, denoted by $p(\mathbf{p})$, to give $\{\mathbf{p}^{(\ell)}; \ell = 1, \ldots, M\}$. The goal of the estimation process is to determine the conditional pdf of the ℓth element $\mathbf{p}^{(\ell)}$ given the current-time measurement $\tilde{\mathbf{y}}_k$. Application of Bayes' rule yields

$$p\left(\mathbf{p}^{(\ell)} \mid \tilde{\mathbf{Y}}_k\right) = \frac{p\left(\tilde{\mathbf{Y}}_k \mid \mathbf{p}^{(\ell)}\right) p\left(\mathbf{p}^{(\ell)}\right)}{\sum_{j=1}^{M} p\left(\tilde{\mathbf{Y}}_k \mid \mathbf{p}^{(j)}\right) p\left(\mathbf{p}^{(j)}\right)} \tag{5}$$

where $\tilde{\mathbf{Y}}_k$ denotes the sequence $\{\tilde{\mathbf{y}}_0, \tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_k\}$. The a posteriori probabilities can be computed through [18]

$$p\left(\mathbf{p}^{(\ell)} \mid \tilde{\mathbf{Y}}_k\right) = \frac{p\left(\tilde{\mathbf{y}}_k, \mathbf{p}^{(j)} \mid \tilde{\mathbf{Y}}_{k-1}\right)}{p\left(\tilde{\mathbf{y}}_k \mid \tilde{\mathbf{Y}}_{k-1}\right)} = \frac{p\left(\tilde{\mathbf{y}}_k \mid \mathbf{x}_k^{-(\ell)}\right) p\left(\mathbf{p}^{(\ell)} \mid \tilde{\mathbf{Y}}_{k-1}\right)}{\sum_{j=1}^{M} p\left(\tilde{\mathbf{y}}_k \mid \mathbf{x}_k^{-(j)}\right) p\left(\mathbf{p}^{(j)} \mid \tilde{\mathbf{Y}}_{k-1}\right)} \tag{6}$$

since $p\left(\tilde{\mathbf{y}}_k \mid \tilde{\mathbf{Y}}_{k-1}, \mathbf{p}^{(\ell)}\right)$ is given by $p\left(\tilde{\mathbf{y}}_k \mid \mathbf{x}_k^{-(\ell)}\right)$ in the Kalman recursion. Note that the denominator of Eq. (6) is just a normalizing factor to ensure that $p\left(\mathbf{p}^{(\ell)} \mid \tilde{\mathbf{Y}}_k\right)$ is a pdf. The recursion formula can now be cast into a set of defined weights $\varpi_k^{(\ell)}$, so that

$$\varpi_k^{(\ell)} = \varpi_{k-1}^{(\ell)} p\left(\tilde{\mathbf{y}}_k \mid \mathbf{x}_k^{-(\ell)}\right), \quad \varpi_k^{(\ell)} \leftarrow \varpi_k^{(\ell)} \Big/ \sum_{j=1}^{M} \varpi_k^{(j)} \tag{7}$$

where $\varpi_k^{(\ell)} \equiv p\left(\mathbf{p}^{(\ell)} \mid \tilde{\mathbf{y}}_k\right)$. The weights at time t_0 are initialized to $\varpi_k^{(\ell)} = 1/M$ for $\ell = 1, 2, \ldots, M$.

The standard MMAE approach runs a set of parallel single-model-based filters, which are independent of each other. This works well with an unknown structure or parameters but requires no structural or parametric changes. Faults typically do not fall under this concept because the structure or parameters do change as a component or subsystem fails. Several approaches can be used to overcome this difficulty [22]. The most common is the interacting multiple-model (IMM) estimator, which "switches" from one model to another in a probabilistic manner. The switches are modeled by a Markov sequence. Like the MMAE approach the IMM estimator also consists of a bank of model-based filters running in parallel at each cycle. However, the initial estimate at the beginning of each cycle for each filter is a mixture of all most recent estimates from the single-model-based filters, which enables it to effectively take into account the history of the modes without the exponentially growing requirements in computation and storage as required by the optimally derived estimator. This provides a faster and more accurate estimate for the changed system states. Also, the probability of each mode is calculated, which indicates the affected mode and transition at each time.

The four major steps in the IMM cycle are: 1) model-conditional re-initialization (interacting or mixing of the estimates), in which the input to the filter matched to a certain mode is obtained by mixing the estimates of all filters at the previous time under the assumption that this particular mode is in effect at the present time; 2) model-conditional filtering, performed in parallel for each mode; 3) mode probability update, based on the model-conditional likelihood functions; and 4) estimate combination, which yields the overall state estimate as the probabilistically weighted sum of the updated state estimates of all filters. The mode probability is provided by the weights used to update the state estimate, which is similar to the MMAE structure.

C. Gaussian Sum Filter

The EKF and UKF work with nonlinear systems and measurement models. The posterior probability density (pdf) function of the vector is still assumed to be represented by a Gaussian distribution. Hence, only the mean and covariance are needed to be maintained and updated in these filters. For nonlinear systems the posterior pdf may not be Gaussian though, which may lead to problems in the EKF and UF. The goal of Gaussian sum filters (GSFs) is to determine the posterior pdf using a sum of Gaussian distributions.

Consider $\tilde{\mathbf{Y}}_k = \left\{\tilde{\mathbf{y}}_0, \tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_k\right\}$, which is the set of measurements up to and including t_k and a state $\mathbf{x}_k$. The Gaussian sum approximation uses a Bayesian estimation approach to construct $p\left(\mathbf{x}_k \mid \tilde{\mathbf{Y}}_k\right)$. The central idea in a GSF is to use a finite set of Gaussian distributions to estimate the pdf $p\left(\mathbf{x}_k \mid \tilde{\mathbf{Y}}_k\right)$. Consider the following Gaussian distribution:

$$N\left(\mathbf{x}^{(j)} \mid P^{(j)}\right) = \frac{1}{\left[\det\left(2\pi P^{(j)}\right)\right]^{1/2}} \exp\left[-\frac{1}{2}\left(\mathbf{x} - \mathbf{x}^{(j)}\right)^T \left(P^{(j)}\right)^{-1} \left(\mathbf{x} - \mathbf{x}^{(j)}\right)\right] \tag{8}$$

where $\mathbf{x}^{(j)}$ is the mean and $P^{(j)}$ is the covariance. The Gaussian approximation is based on the lemma that any probability density $p(\mathbf{x})$ can be approximated by

$$p\left(\mathbf{x}\right) \approx \sum_{j=1}^{N} w_j N\left(\mathbf{x}^{(j)} \mid P^{(j)}\right) \tag{9}$$

for some N and positive weights with $\sum_{j=1}^{N} w_j = 1$, which is required so that the approximated $p(\mathbf{x})$ is indeed a valid pdf.

For nonlinear systems we wish to employ a bank of EKFs in a GSF setting to estimate $p\left(\mathbf{x}_k \mid \tilde{\mathbf{Y}}_k\right)$. A GSF is similar to a MMAE approach. However, in the GSF there is only one random variable, $\mathbf{x}_k$, that is to be estimated, while in the MMAE approach there are multiple random variables associated with each model. Fortunately, the derivation of the update law for the weights in the GSF follows from the theory of the MMAE approach. The table below summarizes the EKF-based GSF.

Gaussian Sum Filter Steps
Model $\mathbf{x}_{k+1} = \mathbf{f}\left(\mathbf{x}_k\right) + \Psi\left(\mathbf{x}_k\right)\mathbf{w}_k, \quad \mathbf{w}_k \sim N\left(\mathbf{0}, Q_k\right)$ $\tilde{\mathbf{y}}_k = \mathbf{h}\left(\mathbf{x}_k\right) + \mathbf{v}_k, \quad \mathbf{v}_k \sim N\left(\mathbf{0}, R_k\right)$
Initialize $\hat{\mathbf{x}}\left(t_0\right) \sim N\left(\mathbf{x}_0^{(j)}, P_0^{(j)}\right)$
Gain $K_k^{(j)} = P_k^{-(j)} H_k^{(j)} \left(E_k^{-(j)}\right)^{-1}$ $E_k^{-(j)} = H_k^{(j)} P_k^{-(j)} H_k^{(j)T} + R_k, \quad H_k^{(j)} \equiv \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right\|_{\hat{\mathbf{x}}_k^{-(j)}}$
Update $\hat{\mathbf{x}}_k^{+(j)} = \hat{\mathbf{x}}_k^{-(j)} + K_k^{(j)} \mathbf{e}_k^{-(j)}, \quad \mathbf{e}_k^{-(j)} \equiv \tilde{\mathbf{y}}_k - \mathbf{h}\left(\hat{\mathbf{x}}_k^{-(j)}\right)$ $P_k^{+(j)} = \left[I - K_k^{(j)} H_k^{(j)}\right] P_k^{-(j)}$
Propagation $\hat{\mathbf{x}}_{k+1}^{-(j)} = \mathbf{f}\left(\hat{\mathbf{x}}_k^{+(j)}\right)$ $P_{k+1}^{-(j)} = \Phi_k^{(j)} P_k^{+(j)} \Phi_k^{(j)T} + \Psi_k^{(j)} Q_k \Psi_k^{(j)T}, \quad \Phi_k^{(j)} \equiv \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right\|_{\hat{\mathbf{x}}_k^{+(j)}}$
Weights $w_k^{(j)} = w_{k-1}^{(j)} p\left(\tilde{\mathbf{y}}_k \mid \hat{\mathbf{x}}_k^{-(j)}\right), \quad w_k^{(j)} \leftarrow w_k^{(j)} \Big/ \sum_{j=1}^{N} w_k^{(j)}$

where the likelihood function is given by

$$p\left(\tilde{\mathbf{y}}_k \mid \hat{\mathbf{x}}_k^{-(j)}\right) = \frac{1}{\left[\det\left(2\pi E_k^{-(j)}\right)\right]^{1/2}} \exp\left\{-\frac{1}{2}\mathbf{e}_k^{-(j)T}\left(E_k^{-(j)}\right)^{-1}\mathbf{e}_k^{-(j)}\right\} \tag{10}$$

The weights at time t_0 are initialized to $1/N$. A set of N initial conditions for the state and covariance for each filter are developed. Note that no filters can be duplicated with the same initial conditions because this will produce identical filters that are redundant. Each filter can have the same covariance but must have different initial states, likewise each filter can have the same initial state but must have different covariances. Extended Kalman filters are executed using the different initial conditions, running through the normal update and propagation stages. The conditional mean estimate is the weighted sum of the parallel filter estimates:

$$\hat{\mathbf{x}}_k^{+} = \sum_{j=1}^{N} w_k^{(j)} \hat{\mathbf{x}}_k^{+(j)} \tag{11}$$

Also, the covariance of the state estimate can be computed using

$$P_k^{+} = \sum_{j=1}^{N} w_k^{(j)} \left[\left(\hat{\mathbf{x}}_k^{+(j)} - \hat{\mathbf{x}}_k^{+}\right)\left(\hat{\mathbf{x}}_k^{+(j)} - \hat{\mathbf{x}}_k^{+}\right)^T + P_k^{+(j)}\right] \tag{12}$$

Reference [20] presents a GSF to characterize the uncertainty associated with orbital tracking problems. For stochastic continuous dynamic systems the exact evolution of the state pdf is given by the Fokker-Planck-Kolmogorov Equation (FPKE). Reference [23] develops an adaptive Gaussian sum filter approach for accurate uncertainty propagation through nonlinear dynamical systems while incorporating the solution to the FPKE.

D. Particle Filter (PF)

Particle filters (PFs) have gained much attention in recent years. Like other approximate nonlinear filtering methods, the ultimate objective of the PF is to reconstruct the posterior pdf of the state vector, or the probability distribution of the state vector conditional on all the available measurements. However, the approximation of the PF is vastly different from that of conventional nonlinear filters. By approximating a continuous distribution of interest by a finite (but large) number of weighted random samples or particles in the state space, the PF assumes no functional form for the posterior probability distribution. In the simplest form of the PF, the particles are propagated through the dynamic model and then weighted according to the likelihood function, which determines how closely the particles match the measurements. Those that best match the measurements are multiplied and those that do not are discarded.

In principle, the PF (with an infinite number of particles) can approximate the posterior probability distribution of any form and solve any nonlinear and/or non- Gaussian estimation problem. In practice, however, it is nontrivial to design a PF with a relatively small number of particles. The performance of the PF heavily depends on whether the particles are located in the significant regions of the state space and whether the significant regions are covered by the particles. When the measurements are accurate, which is typical for many estimation problems, the likelihood function concentrates in a small region of the state space, and the particles propagated through the dynamic model are more often than not located outside the significant regions of the likelihood function. State estimates such as the mean and covariance approximated with these particles are imprecise. This problem becomes even worse when the initial estimation errors are large, for example, a few orders of magnitude larger than the sensor accuracy. Consequently, the basic PF quickly suffers the problem of severe particle degeneracy (the loss of diversity of the particles) and filter divergence.

The particles of the PF are randomly sampled from an importance function. The importance weight associated with each particle is adaptively computed based on the ratio between the posterior pdf and the importance function (up to a constant). Given the particles, higher moments of interest as well as the mean and covariance can be computed in a straightforward manner whenever desired. From these particles, it is also convenient to compute statistics such as the modes and the median, which may be desired in certain applications. Stated in another way, the PF provides a whole picture of the underlying distribution.

The bootstrap filter (BF) was first derived by Gordon, Salmond and Smith [9]. Being the first operational PF, the BF is modular and easy to implement. The justification for the BF is based on asymptotic results. Thus, it is usually difficult to prove any general result for a finite number of samples or to make any precise, provable statement on how many samples are required to give a satisfactory representation of the pdf. We prefer to use as few particles as possible in the BF, because the computational cost of the BF is largely proportional to the number of particles. For a BF with a modest number of particles to work properly, the sampling efficiency has to be enhanced. In order to do this, in the proposed BF the scheme of particle roughening is typically used. More details on PFs can be found in Ref. [8].

E. Probability Hypothesis Density (PHD) Filter

Note that all aforementioned non-linear filters are capable of addressing the MTMS design challenges discussed in Section 1.0; however, the data association at the front end and the track fusion at the back end still have to be handled in an ad-hoc or traditional fashion. In addition, the second major cumbersome implementation associated with those nonlinear filters is that for every single object to be tracked and estimated on the track file side, a nonlinear filter has to be implemented in parallel. As a result, for 10,000 of objects or beyond for the space traffic modeling and tracking and control, 10,000 of nonlinear filters have to be implemented and maintained/processed in parallel. This implementation issue when coupled with the front end measurement to non linear filter association (for correct measurement to be used by respective nonlinear filter at each measurement update) will generate a tremendous amount of permutations and computational loads during the gating process (See detailed discussion Section 4). Likewise, by the same token track merging at the output of each nonlinear filter in the back end needs to be addressed as well to ensure track life of RSOs' state vectors to be properly managed and maintained.

It is only recently that a rigorous mathematical framework has been established [Mahler] allowing us to handle the multiple object tracking problem by just using a single meta filter via the finite set statistics (FISST) [Mahler & Nguyen, Vo] with Random Finite Set (RFS) as the "multiple object" set estimate whose elements are the individual respective RSOs state vectors. First the collection of targets and the collection of observations are both treated as separate *set-valued entities*. These set-valued entities are then modeled by separate RFSs s that the problem of multi-target tracking can be formulated in the Bayesian frame-work. This has led to the development of novel and efficient multi-target filters and their computationally efficient approximations, which have generated substantial interests and set a stage for multiple development directions.

How the RFS and PHD filter framework can be leveraged to turn a complex multiple target multiple sensor system into a *"single meta target" and "single meta sensor"* is discussed next. This **single meta target** is possible thanks to the random finite set framework which practically captures all individual targets or objects in a **random set** regardless of their actual population value (i.e., several hundred thousand or beyond). This single meta target random set is then being process by a single PHD filter to predict the movement of all objects captured in that random set. Likewise, the **single meta sensor** is also being captured by a **random (sensors) set**. In other words, all sensors are now being "stacked" in a random set which contains multiple target measurements collected by multiple sensors. Users who are interested in this great strategy and attractive framework are recommended to check into work done by Mahler (Refs. 10 & 16) and Vo & Vo (Refs. 11 & 13).

Multi-target state and multi-target measurement at time k are naturally represented as finite sets X_k and Z_k. For example, if at time k there are $M(k)$ targets located at

$$\{x_k^1, x_k^2, \dots x_k^M\} \in E_S \tag{13}$$

then,

$$X_k = [\, x_k^1, x_k^2, \dots, x_k^M] \subseteq F(E_S) \tag{14}$$

Note that X_k is now a set representation and $F(E_S)$ is the collection of all finite subsets of E_S

Similarly, if $N(k)$ observations at time k,

$$\{z_k^1, z_k^2, \dots z_k^N\} \in E_O \tag{15}$$

are received at time k, then

$$Z_k = [\, z_k^1, z_k^2, \dots z_k^N] \subseteq F(E_O) \tag{16}$$

where some of the $N(k)$ observations may be due to clutter.

Analogous to single target system, where uncertainty is characterized by modeling the state and measurement by random vectors, uncertainty in a multi-target system is characterized by modeling multi-target state and multi-target measurement as random finite sets (RFS) Ξ_k and Σ_k on the state and observation spaces E_s and E_o, respectively.

Given a realization X_{k-1} of the multi-target state at time k–1, the multi-target state at time k can be modeled by the RFS

$$\Xi_k = S_k(X_{k-1}) U\, B_k(X_{k-1}) U\, \Gamma_k \tag{17}$$

where $S_k(X_{k-1})$ denotes the RFS of targets that have survived at time k, $B_k(X_{k-1})$ denotes the RFS of targets spawned from X_{k-1} and Γ_k denotes the RFS of targets that appear spontaneously at time k.

Likewise, for measurement set it accounts for (i) actual target measurement; (ii) clutter; and (iii) sensor noise

$$\Sigma_k = \mathrm{E}_k(\mathrm{X}_{k-1})\mathrm{U}\ \mathrm{C}_k(\mathrm{X}_{k-1})\mathrm{U}\ \mathrm{M}_k \tag{18}$$

The three sets (being accounted for as a union of all) on the right hand side of equations (18) represent for sensor measurement, clutter, and sensor noise, respectively.

Important Note: Equation (17) is the key RFS modeling of the target space illustrating how target appearance, death, and spawning can be captured in the target space Ξ_k. This target set space with its multiple targets existence therein later will be integrated using the RFS target density $D_{\Xi(k)}$ to pinpoint exactly how many targets are residing in that target space in real time regardless the target types (i.e., birth, death, or spawning).

The following section presents a tractable solution of the PHD filter using the Sequential Monte Carlo (SMC) implementation. Via this PF based SMC implementation, the PHD filter yields an attractive multiple target detection and tracking solution which is highly applicable to the space traffic modeling, management, and control.

III. PF Based Design for PHD Filter Implementation

The PHD filter can be implemented using the PF based filter design techniques discussed in Section II. SMC implementation (over Gaussian Sum Filter or Gaussian Mixture) is employed as an illustration example in this survey.

A. SMC Implementation Discussion

The SMC implementation scheme has been recognized as a popular means for implementation of a PHD filter. Detailed SMC implementation process can be obtained by cross checking with References [10], [11], and [13]. Therefore, it is not repeated herein.

B. Simulation Results and Discussion

Figures 1 to 4 present the performance of the SMC-PHD filter

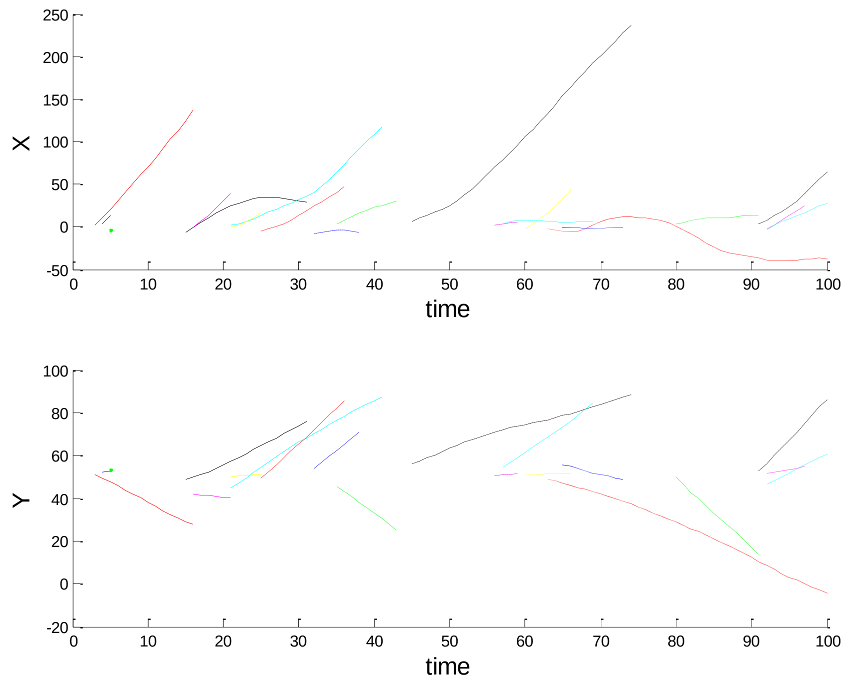

Figure 1: True Multiple Targets' Tracks in X-Y Coordinate

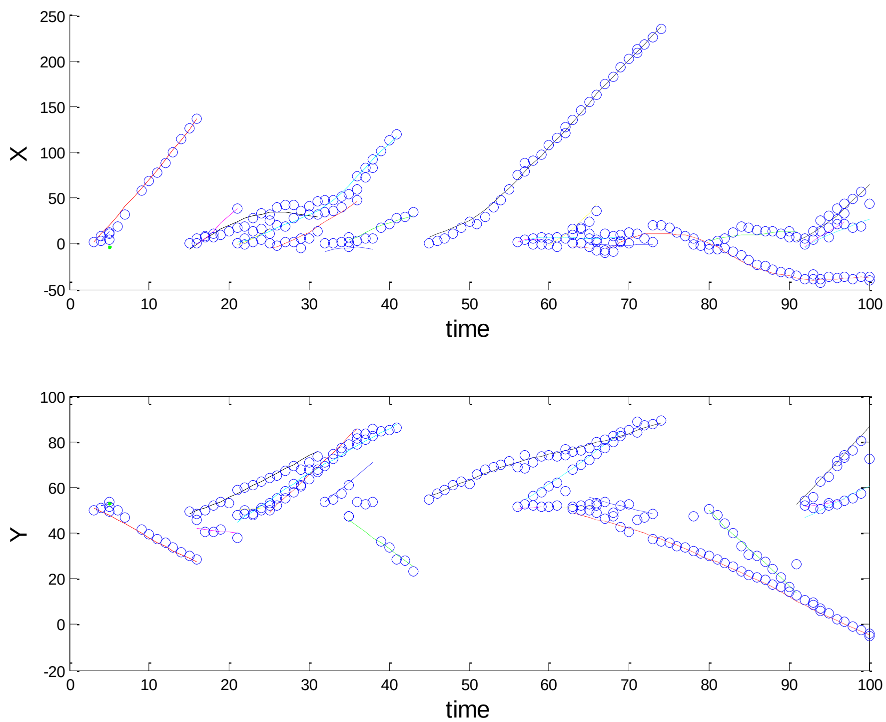

Figure 2: True Multiple Targets' Tracks vs Multiple Target's Estimated Track

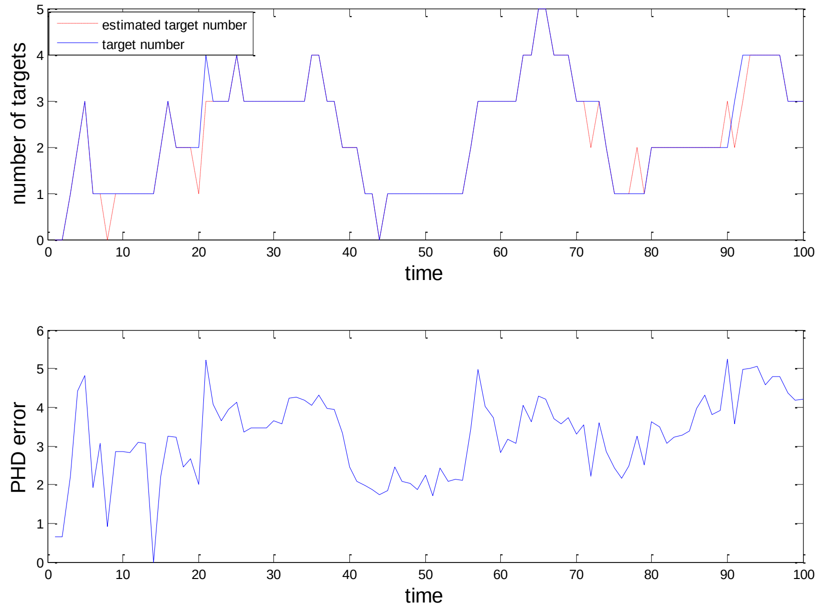

Figure 3: Estimated Number of Targets vs True Number of Varying Targets & Its PHD Error

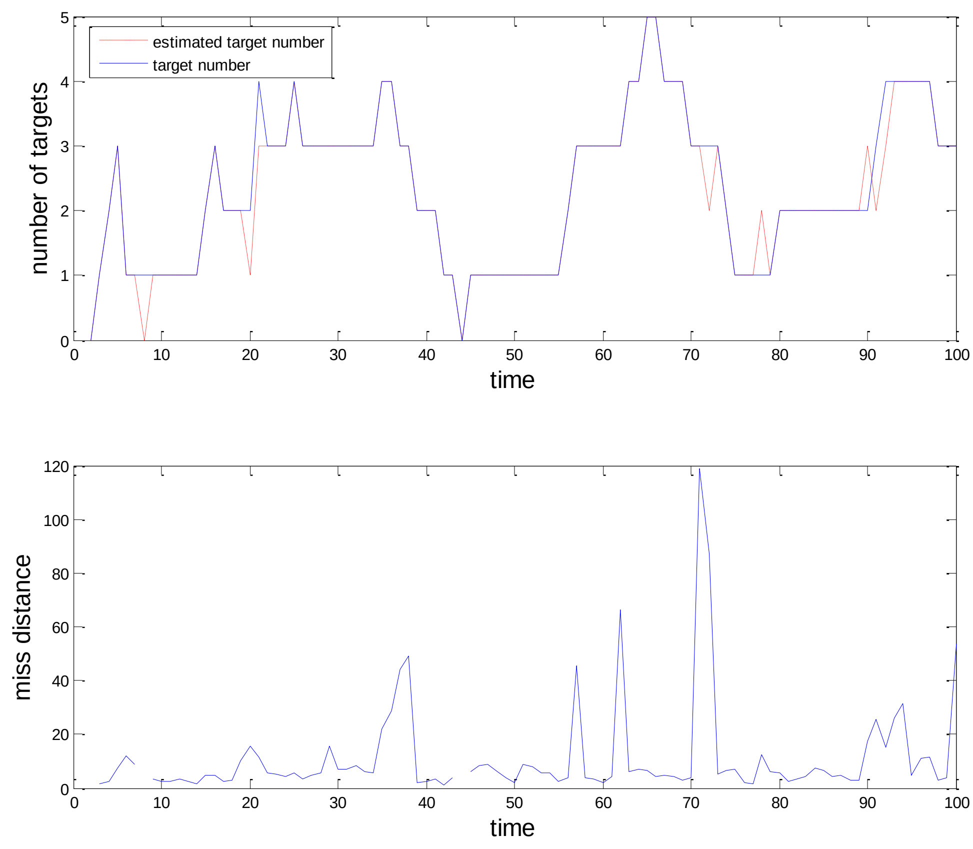

Figure 4: SMC-PHD Performance in Estimating the Number of Targets in Real Time

IV. Benefits of PHD Filter & Its Future Development Directions

The direct benefit of the PHD/RFS design framework is that the front end measurement to object's state vector estimate (i.e., object's track under the traditional MTMS design framework) is no longer needed (see Figure 5 for front end measurement to track data association). We use the Global Nearest Neighbor (GNN) filtering scheme depicted in Figure 5 to illustrate this benefit. The PHD/RFS framework can produce multiple objects' state vector estimates without explicitly performing the measurement to object's state vector estimate association (i.e., measurement to track fusion or association), it will eliminate a tremendous amount of computational load and logical check upfront when the object population exceeds 10,000 objects. The other benefit of the PHD/RFS framework is its ability to quickly converge to the "truth" state vector without heavily relying on the state vector's initial condition of a new object (i.e., initial orbit determination.)

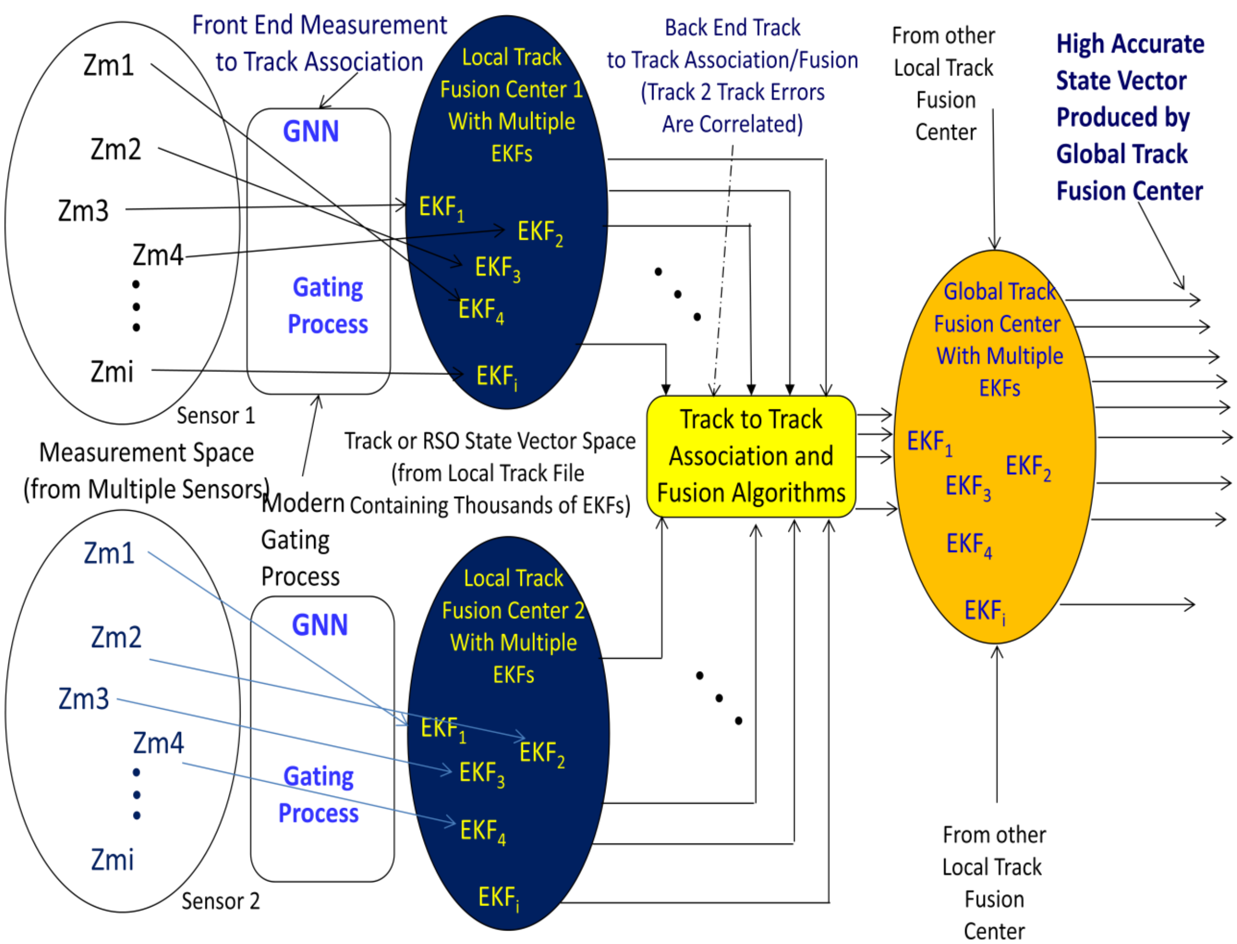

Figure 5: GNN Tracking Architecture With Front End Data Association

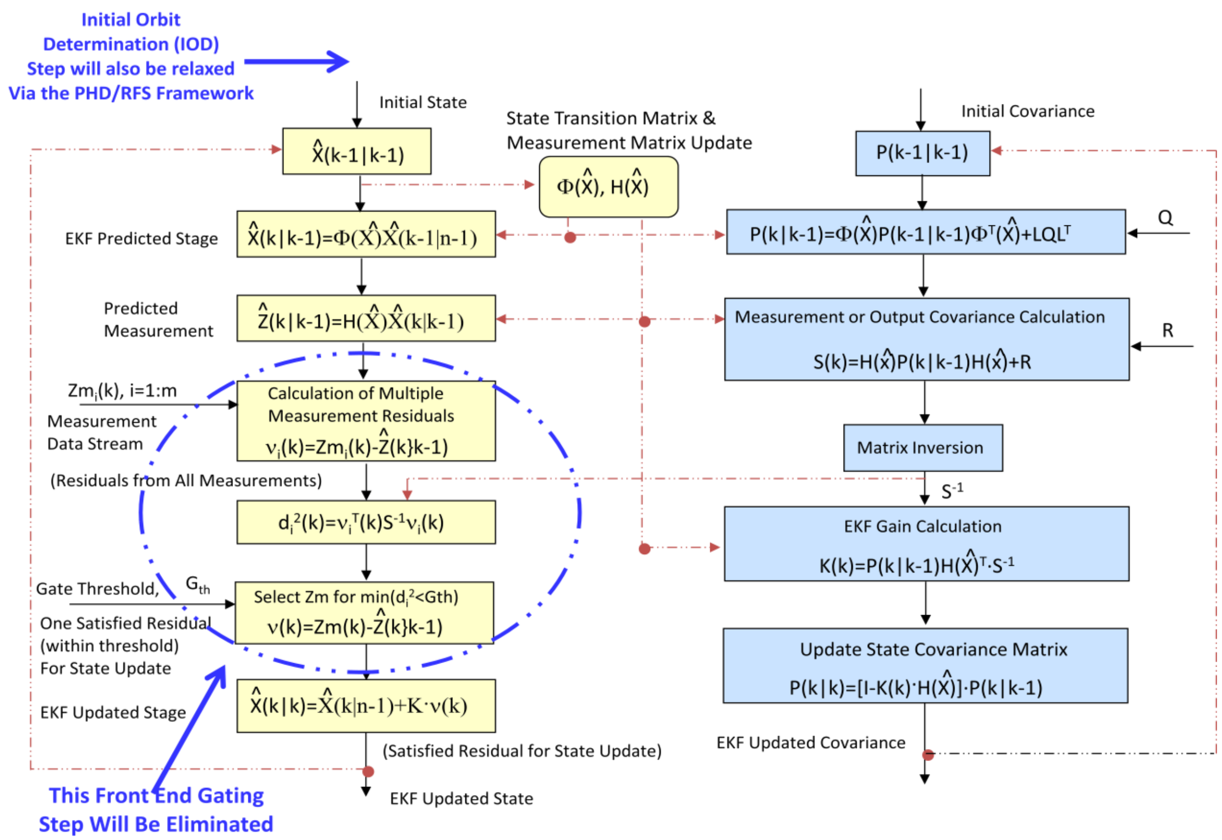

Figure 6: GNN Gating Process & Single EKF Processing Flow

V. Conclusion

Nonlinear estimation problems, especially in the context of multiple target multiple sensor tracking paradigm, are inherent challenging topic for both theorists and practitioners to iron out a path in mixing R&D solutions with practical implementation techniques to meet their emerging needs. While reality is still dwelling on EKF based and ad-hoc utilization of UKF, IMM/JPDA, and PF based, the PHD/RFS framework does offer a viable path to resolve the computational intensive front end data association (i.e., ten thousands of object measurements to be associated with the right current object's state vector for update and prediction via a gating process). However, its applicability to effectively solve the back end of the object's state vector fusion (i.e., track to track fusion) remains to be seen. It is quite a challenge to address track fusion within the PHD/RFS framework; however, an emerging trend toward reaching a practical solution is observed to be reachable (e.g., see [27] by Panta and [28] by Clark). With the number of targets approaching 100,000 range, a judicious selection of various schemes may be the right way to cure this high target population.

Acknowledgments

The first author, Quang Lam, would like to express his sincere appreciation to Dr. B. N. Vo for his insight and useful discussion on the payoffs of RFS/PHD based filter design paradigm.

References

[1]Y. Bar-Shalom and T. E. Fortmann, ***Tracking and Data Association***, Academic Press, 1988

[2]Y. Bar-Shalom & X. R. Li, **Estimation and Tracking: Principles, Techniques, and Software**, Boston, MA: Artech House, 1993

[3]X. R. Li, "Multiple-Model Estimation with Variable Structure: Some Theoretical Considerations," Proceedings of the 1994 CDC.

[4]Y. Bar-Shalom and W. D. Blair, **MultiTarget-MultiSensor Tracking – Applications and Advances**, Artech House Inc., 2000
[5]Q. Lam, S. Fujikawa, X. R. Li, "Future Trends to Enhance the Robustness of a Target Tracker," Presented at the AIAA GN&C Conference in San Diego, CA July 1996
[6]Q. M. Lam and J. L. Crassidis, "Evaluation of a Multiple Model Adaptive Estimation Scheme for Space Vehicle's Enhanced Navigation Solution," Presented at *the AIAA GN&C August 2007, Hilton Head, SC*
[7]Alsuwaidan, B.N., Crassidis, J.L., and Cheng, Y., "Generalized Multiple-Model Adaptive Estimation Using an Autocorrelation Approach," *IEEE Transactions on Aerospace and Electronic Systems*, accepted for publication.
[8]Crassidis, J.L. and Junkins, J.L., *Optimal Estimation of Dynamic Systems*, Second Edition, Chapman & Hall/CRC, Boca Raton, FL, 2012.
[9]N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," *Proc. Inst. Elect. Eng. F*, vol. 140, pp. 107–113, 1993.
[10]R. Mahler, **Statistical Multisource-Multitarget Information Fusion**. Artech House, 2007
[11]B. N. Vo, Random Finite Sets in Stochastic Filtering, IEEE Victorian Chapter July 28, 2009, University of Melbourne, Australia
[12]D. Schuhmacher, B.-T. Vo, and B.-N. Vo, "A consistent metric for performance evaluation in multi-object filtering," IEEE Trans. Signal Processing, Vol. 56, No. 8 Part 1, pp. 3447– 3457, 2008
[13]B.-T. Vo, ***Random Finite Sets in Multi-Object Filtering***, Ph. D. Dissertation, University of Western Australia, October 2008
[14]Goodman I., Mahler R., and Nguyen H., ***Mathematics of Data Fusion***, Kluwer Academic Publishers, 1997
[15]Goutsias J., Mahler R., and Nguyen H. (eds.), ***Random Sets Theory and Applications***, Springer-Verlag New York, 1997.
[16]Mahler R., "Multi-target moments and their application to multi-target tracking," *Proc. Workshop on Estimation, Tracking and Fusion*: A tribute to Yaakov Bar-Shalom, Monterey, pp. 134-166, 2001
[17]R. Mullikin, "*Counterspace Capabilities using Small Satellites: Bridging the Gap in Space Situational Awareness," 6th Annual Disruptive Technologies Conference, October 2009*
[18]Volkan Cevher, Rajbabu Velmurugan, and James H. McClellan, *"*A Range-Only Multiple Target Particle Filter Tracker," Georgia Technical Institute Report, U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-02-0008.
[19]I. Hwang, H. Balakrishnan, K. Roy, and C. Tomlin, "Multiple Target Tracking and Identity Management with Application to Aircraft Tracking," Journal of Guidance, Control, and Dynamics, No.3, 2007
[20]J. T. Horwood, N. D. Aragon, and A. B. Poore, "Gaussian Sum Filters for Space Surveillance Theory and Simulations," Journal of Guidance, Control, and Dynamics, No.6, 2011.
[21]B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, chap. 10.1, Dover Publications, Mineola, NY, 2005.
[22]Bar-Shalom, Y., Li, X.R., and Kirubarajan, T., *Estimation with Applications to Tracking and Navigation*, John Wiley & Sons, New York, NY, 2001.
[23]Terejanu, G., Singla, P., Singh, T., and Scott, P. D., "Uncertainty Propagation for Nonlinear Dynamical Systems using Gaussian Mixture Models," Journal of Guidance, Control, and Dynamics, Vol. 31, 2008, pp. 1623-1633.
[24]Joseph J. LaViola Jr. "A Comparison of Unscented and Extended Kalman Filtering for Estimating Quaternion motion" 2003
[25]Simon J. Julier Jeffrey K. Uhlmann, "A New Extension of the Kalman Filter to Nonlinear Systems" .1997
[26]Lam, Q. M., Junker, D., Anhalt, D., and Vallado, D., "Analysis of an Extended Kalman Filtering Based Orbit Determination System," Presented at the AIAA GN&C Conference, Toronto, Canada, August 2010
[27]K. Panta, **Multiple Target Tracking Using 1st Moment of Random Finite Sets**, Ph. D. Dissertation, University of Melbourne, 2007
[28]D. E. Clark, **Multiple Target Tracking with the Probability Hypothesis Density Filter**, Ph. D. Dissertation, Heriot-Watt University, 2006