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CHOOSING FILTER STATES AND MODELS FOR SMALL SATELLITE ATTITUDE DETERMINATION

Andrew D. Dianetti*[*] and John L. Crassidis[†]

The availability of low-cost attitude sensors, including gyroscopes, has enabled precision attitude determination and control in small satellites. Kalman filtering is one of the most popular methods for such determination. Historically, dynamic model replacement has been used for angular rate estimates, where only the gyro bias is estimated, and the angular rate estimate is obtained by subtracting it from the gyro reading. However, the comparatively high noise in sensors used in small satellites results in reduced performance when this formulation is used. Estimating angular rate in the filter allows for better angular rate estimates, but disturbance torques can no longer be ignored in the model. This work presents filters for both the case of dynamic model replacement and estimating the angular rate, and simulations are presented for several use cases. It is found that for the typical sensor noise parameters present in small satellites, estimating angular rate in the filter results in better estimates, especially of the angular rate state.

INTRODUCTION

Kalman filtering is one of the most popular methods for spacecraft attitude determination. The Extended Kalman Filter (EKF) has historically been one of the most commonly used methods. Other recursive methods, such as the Unscented Filter and Filter QUEST, are also used.[1] Low-cost Microelectromecchanical Systems (MEMS) sensors, such as magnetometers and gyroscopes, have enabled precision attitude determination in small satellites. However, these sensors have high noise parameters in comparison to sensors traditionally used on spacecraft.

Traditional implementation of the recursive methods in large satellites has used dynamic model replacement, in which the estimated states consist of the attitude and gyroscope bias. Since random noise in conventional gyros is low, subtracting the estimated bias from the gyro reading produces an accurate measure of the true angular rate, and this can be fed directly into the filter. Such a method eliminates the need for knowledge of the physical model, including the inertia matrix and commanded torques, and also captures disturbance torques. Many proposed and implemented small satellite attitude determination and control systems using gyroscopes also employ this method.[2–5] However, MEMS gyroscopes have noise that is several orders of magnitude higher than conventional or ring laser gyroscopes. In order to use the dynamic replacement model, this noise must be accounted for in the process noise covariance, which leads to an increased uncertainty in the state estimate. Since the angular rate estimate is determined directly from the gyroscope measurements, the angular rate estimate becomes particularly noisy. Using a full-state estimator that estimates attitude, rate, and gyro bias can result in greater accuracy by removing this source of process noise. This is especially true if a star tracker or other high-accuracy attitude sensor is used, as the angular rate estimates will also be updated from information contributed by this sensor. However, this formulation requires knowledge of the attitude dynamics, and requires treatment of unknown disturbance torques as process noise.

*Graduate Student, Department of Mechanical & Aerospace Engineering, University at Buffalo, State University of New York, Amherst, NY 14260. Student Member AAS. Email: andrewdi@buffalo.edu

†CUBRC Professor in Space Situational Awareness, Department of Mechanical & Aerospace Engineering, University at Buffalo, State University of New York, Amherst, NY 14260. Fellow AAS. Email: johnc@buffalo.edu

This paper will present formulations of the Multiplicative EKF and Unscented Filter using both dynamic model replacement and a full-state estimator. Simulations will be presented for different classes of sensor accuracy, and the respective performance of each filter will be discussed.

ATTITUDE DYNAMICS AND REPRESENTATIONS

Quaternion Kinematics

Although a spacecraft's attitude is described by three degrees of freedom, there exist no nonsingular three parameter representations. The quaternion, a four-parameter realization, is one of the most commonly used representations in attitude determination. It is composed of a 3×1 vector $\boldsymbol{\varrho}$ and a scalar q_4:

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{\varrho}^T & q_4 \end{bmatrix}^T \tag{1}$$

The quaternion is subject to the constraint $||\boldsymbol{q}|| = 1$. Unless otherwise noted, the quaternion describes the rotation from the inertial frame to the spacecraft body frame.

The spacecraft's angular rate, measured with respect to an inertial frame but written in body-frame coordinates, is denoted $\boldsymbol{\omega}$, a 3×1 vector. Then, the quaternion kinematics are given by[6]

$$\dot{\boldsymbol{q}} = \frac{1}{2}\Xi(\boldsymbol{q})\boldsymbol{\omega} = \frac{1}{2}\Omega(\boldsymbol{\omega})\boldsymbol{q} \tag{2}$$

where

$$\Xi(\boldsymbol{q}) = \begin{bmatrix} q_4 I_{3\times3} + [\boldsymbol{\varrho}\times] \\ -\boldsymbol{\varrho}^T \end{bmatrix} \tag{3a}$$

$$\Omega(\boldsymbol{\omega}) = \begin{bmatrix} -[\boldsymbol{\omega}\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix} \tag{3b}$$

where $I_{3\times3}$ is the 3×3 identity matrix and $[\boldsymbol{\omega}\times]$ denotes the skew-symmetric cross product matrix of $\boldsymbol{\omega}$.

Kalman Filter Attitude State Formulations

Due to the unit norm constraint, the quaternion cannot be used directly in the Kalman filter, as the constraint may be violated. A common approach to using the quaternion in the Extended Kalman Filter and Multiplicative Extended Kalman Filter (MEKF), which maintains the unit norm to first order. Derivations of the and MEKF are widely available.[1,7] The MEKF is widely used due to its simplicity. In the MEKF, the true attitude is written as a product of the estimated attitude and a deviation from the estimate. If the error quaternion is defined as $\boldsymbol{\delta q}$, then the true quaternion is written as

$$\boldsymbol{q} = \boldsymbol{\delta q} \otimes \hat{\boldsymbol{q}} \tag{4}$$

Note that quaternion multiplication $\otimes$ is defined using the Shuster convention, where quaternions are composed in the same order as attitude matrices.[8] Here, the error quaternion $\boldsymbol{\delta q}$ is the deviation from the estimate. It can be written as

$$\boldsymbol{\delta q} = \begin{bmatrix} \boldsymbol{\delta\varrho}^T & \delta q_4 \end{bmatrix}^T \tag{5}$$

The attitude states used in the filter are $\boldsymbol{\delta\varrho}$, and the scalar part of the error quaternion is computed as

$$\delta q_4 = \sqrt{1 - ||\boldsymbol{\delta\varrho}||} \tag{6}$$

In the MEKF, the estimated attitude states are $\boldsymbol{\delta\varrho}$. At each time step k, the error quaternion $\boldsymbol{\delta q}$ is initialized as identity:

$$\boldsymbol{\delta q}_k^- = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{7}$$

Then, the Kalman update is performed on $\boldsymbol{\delta\varrho}$ and the updated error quaternion is written as

$$\boldsymbol{\delta q}_k^+ = \begin{bmatrix} (\boldsymbol{\delta\varrho}_k^+)^T & \delta q_{4,k}^+ \end{bmatrix}^T \tag{8}$$

where $\delta q_{4,k}^{+}$ is computed using Eq. (6). Then, the updated quaternion estimate at each time step is given by applying Eq. (4)

$$\hat{\boldsymbol{q}}_k^{+} = \boldsymbol{\delta q}_k^{+} \otimes \hat{\boldsymbol{q}}_k^{-} \tag{9}$$

Alternatively, if $\boldsymbol{\delta\varrho}$ is small, then a first-order approximation can be made:[6]

$$\hat{\boldsymbol{q}}_k^{+} \approx \left(\boldsymbol{I}_q + \begin{bmatrix} \boldsymbol{\delta\varrho}_k^{+} \\ 0 \end{bmatrix} \right) \otimes \hat{\boldsymbol{q}}_k^{-} = \hat{\boldsymbol{q}}_k^{-} + \boldsymbol{\delta\varrho}_k^{+} \otimes \hat{\boldsymbol{q}}_k^{-} = \hat{\boldsymbol{q}}_k^{-} + \Xi(\hat{\boldsymbol{q}})\boldsymbol{\delta\varrho} \tag{10}$$

where $\boldsymbol{I}_q$ is the identity quaternion. In this approach, the quaternion must be re-normalized after every update step. With the attitude parameterizations of the MEKF now defined, forms of the MEKF will be presented for both the cases of dynamic model replacement and a full-state filter including angular rate.

DYNAMIC MODEL REPLACEMENT FILTERS

In the dynamic model replacement formulation, a sensor must be available that measures the angular rate. If only attitude, not rate, sensors are available, the angular rate must be estimated. A common noise model for the gyro is[9]

$$\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} + \boldsymbol{\beta} + \boldsymbol{\eta}_v \tag{11a}$$

$$\dot{\boldsymbol{\beta}} = \boldsymbol{\eta}_u \tag{11b}$$

where $\tilde{\boldsymbol{\omega}}$ is the gyro measurement, $\boldsymbol{\beta}$ is the bias vector, and $\boldsymbol{\eta}_v$ and $\boldsymbol{\eta}_u$ are zero-mean Gaussian processes with spectral densities $\sigma_v^2 I_{3\times3}$ and $\sigma_u^2 I_{3\times3}$, respectively. The dynamic model replacement form of the filter leverages the fact that $\boldsymbol{\eta}_v$ is very small for most gyroscopes, and the angular rate estimate is determined by estimating the bias vector and subtracting it from the gyro readings:

$$\hat{\boldsymbol{\omega}} = \tilde{\boldsymbol{\omega}} - \hat{\boldsymbol{\beta}} \tag{12}$$

where $\hat{\boldsymbol{\omega}}$ is the angular rate estimate and $\hat{\boldsymbol{\beta}}$ is the bias estimate. This formulation has the advantage that it does not require knowledge of command torques, and does not require process noise compensation for unknown disturbance torques. However, process noise compensation is required in the attitude state to account for the gyro noise, as will be shown.

Multiplicative Extended Kalman Filter

The estimated states in the filter are the attitude and the bias vector. A full derivation of this filter can be found in (Reference 10), and only its key components are summarized here for brevity. The attitude is represented using the form of the MEKF presented earlier, except the attitude state is represented as $\boldsymbol{\delta\alpha}$, where

$$\boldsymbol{\delta\alpha} = 2\boldsymbol{\delta\varrho} \tag{13}$$

This formulation is chosen as it simplifies the attitude kinematics. Additionally, $\boldsymbol{\delta\alpha}$ has physical significance as errors in roll, pitch, and yaw. Defining the error in the angular rate and bias estimates to be

$$\boldsymbol{\delta\omega} \equiv \boldsymbol{\omega} - \hat{\boldsymbol{\omega}} \tag{14a}$$

$$\boldsymbol{\Delta\beta} \equiv \boldsymbol{\beta} - \hat{\boldsymbol{\beta}} \tag{14b}$$

and substituting Equations (11a) and (12) yields

$$\boldsymbol{\delta\omega} = -(\boldsymbol{\Delta\beta} + \boldsymbol{\eta}_v) \tag{15}$$

Substituting Equations (13) and (15) into the quaternion kinematics given in Eq. (2) gives the linearized error-state kinematics:

$$\boldsymbol{\delta\dot{\alpha}} = -[\boldsymbol{\omega}\times]\boldsymbol{\delta\alpha} - (\boldsymbol{\Delta\beta} + \boldsymbol{\eta}_v) \tag{16}$$

The EKF state vector is given by

$$\boldsymbol{\Delta x}(t) = \begin{bmatrix} \boldsymbol{\delta\alpha}(t)^T & \boldsymbol{\Delta\beta}(t)^T \end{bmatrix}^T \tag{17}$$

and the linearized error model is given by

$$\boldsymbol{\Delta}\dot{\boldsymbol{x}}(t) = F(t)\boldsymbol{x}(t) + G(t)\boldsymbol{w}(t) \tag{18}$$

where

$$\boldsymbol{w}(t) = \begin{bmatrix} \boldsymbol{\eta}_v(t)^T & \boldsymbol{\eta}_u(t)^T \end{bmatrix}^T \tag{19}$$

and

$$F(t) = \begin{bmatrix} -[\hat{\boldsymbol{\omega}}(t)\times] & -I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \tag{20a}$$

$$G(t) = \begin{bmatrix} -I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{20b}$$

Since the gyro readings appear directly in the measurement model, the gyro noise $\boldsymbol{\eta}_v$ appears as process noise in the attitude state. The process noise covariance matrix is written as

$$Q(t) = \begin{bmatrix} \sigma_v^2 I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \sigma_u^2 I_{3\times3} \end{bmatrix} \tag{21}$$

Attitude measurements are typically unit vector measurements, for example from a star tracker, sun sensor, magnetometer, or earth horizon sensor. If these measurements are denoted $\tilde{\boldsymbol{b}}_1$, $\tilde{\boldsymbol{b}}_2$, ..., $\tilde{\boldsymbol{b}}_n$, the measurement vector at time k is then given by

$$\tilde{\boldsymbol{y}}_k = \begin{bmatrix} \tilde{\boldsymbol{b}}_{1,k}^T & \tilde{\boldsymbol{b}}_{2,k}^T & \dots & \tilde{\boldsymbol{b}}_{n,k}^T \end{bmatrix}^T \tag{22}$$

and the measurements can be modeled as

$$\tilde{\boldsymbol{y}}_k = \begin{bmatrix} (A(\boldsymbol{q})\boldsymbol{r}_1)^T & (A(\boldsymbol{q})\boldsymbol{r}_2)^T & \dots & (A(\boldsymbol{q})\boldsymbol{r}_n)^T \end{bmatrix}^T + \boldsymbol{v}_k \tag{23}$$

where v_k is the measurement noise. The vector measurements are unit vectors, and thus the 3×3 covariance matrix R would be singular. However, Shuster has shown that for the MEKF, this can be replaced with[11]

$$R_k = \text{diag}\begin{bmatrix} \sigma_1^2 I_{3\times3} & \sigma_2^2 I_{3\times3} & \dots & \sigma_n^2 I_{3\times3} \end{bmatrix} \tag{24}$$

The sensitivity matrix for all the measurements is given by

$$H_k(\hat{\boldsymbol{x}}_k^-) = \begin{bmatrix} [A(\hat{\boldsymbol{q}}^-)\boldsymbol{r}_1\times] & 0_{3\times3} \\ [A(\hat{\boldsymbol{q}}^-)\boldsymbol{r}_2\times] & 0_{3\times3} \\ \vdots & \vdots \\ [A(\hat{\boldsymbol{q}}^-)\boldsymbol{r}_n\times] & 0_{3\times3} \end{bmatrix} \tag{25}$$

The standard EKF update and covariance propagation can now be applied.

Unscented Filter

The Unscented Filter propagates the uncertainty in the system forward by creating a distribution of ”sigma points” that capture the mean and covariance of the distribution, and propagating these forward through the model. This eliminates the need for linearization of the dynamics and use of the F and H matrices above. Crassidis and Markley have developed the Unscented Filter for attitude estimation using dynamic model replacement.[12] The key components of this filter as they relate to this work are briefly summarized below.

The quaternion kinematics follow the same form as in Eq. (2). The error state is defined as in Eq. (5). However, unlike in the EKF, estimating the vector part of the error quaternion, $\boldsymbol{\delta\varrho}$, does not work well. If this is chosen as the attitude state, and the sigma point distribution is large, it is possible that the sigma points have $||\boldsymbol{\varrho}|| > 1$. This is not physically realizable, and prohibits the use of Eq. (6). Additionally, the small angle assumption used in Eq. (10) does not hold, and poor filter performance may occur if this quaternion is normalized. As such, a three-parameter representation of the error state is used, which ensures that there are no constraints that can be violated. Note that this approach could also be used with the EKF, as presented in (Reference 6). The chosen parameterization is a vector of generalized Rodrigues parameters for the error state. This is defined as[13]

$$\boldsymbol{p} \equiv f \frac{\boldsymbol{\delta\varrho}}{a + \delta q_4} \tag{26}$$

where a is a parameter from 0 to 1 and f is a scale factor. By choosing $f = 2(a+1)$, $||\boldsymbol{\delta p}||$ is equal to the attitude error $\boldsymbol{\delta\alpha}$ for small errors. The inverse of this relation is given by[13]

$$\delta q_4 = \frac{-a||\boldsymbol{\delta p}||^2 + f\sqrt{f^2 + (1-a^2)||\boldsymbol{\delta p}||^2}}{f^2 + ||\boldsymbol{\delta p}||^2} \tag{27a}$$

$$\boldsymbol{\delta\varrho} = f^{-1}(a + \delta q_4)\boldsymbol{\delta p} \tag{27b}$$

The dynamic model replacement is carried out as in the EKF. Note that since the bias vector is being estimated, and is subtracted from $\tilde{\boldsymbol{\omega}}$ in the propagation, this must be done for each sigma point:

$$\hat{\boldsymbol{\omega}}_k^+(i) = \tilde{\boldsymbol{\omega}}_k - \boldsymbol{\chi}_k^\beta(i) \tag{28}$$

where $\boldsymbol{\chi}_k^\beta(i)$ refers to the bias vector portion of the sigma point i. The standard Unscented Filter method is now employed; further explanation is omitted for brevity.

ANGULAR RATE FILTERS

The dynamic model replacement formulations that have been shown are advantageous in that they do not require knowledge of command torques or process noise compensation for disturbance torques. For gyros with a small σ_v, this can be advantageous. However, MEMS gyros typically have σ_v values that are several orders of magnitude higher than traditional mechanical gyros. Since σ_v appears as process noise in the attitude state when dynamic model replacement is used, this can lead to poor angular rate estimates, and subsequent loss of attitude accuracy between attitude measurements, even if a high-accuracy attitude sensor such as a star tracker is used. This can be limiting in applications requiring precision-pointing applications. Utilizing a full-state filter with the estimated state vector containing

$$\hat{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{\delta\alpha}^T & \hat{\boldsymbol{\omega}}^T & \hat{\boldsymbol{\beta}}^T \end{bmatrix}^T \tag{29}$$

can yield better results. This section presents forms of the EKF and Unscented Filter that make use of angular rate as an estimated state.

Multiplicative Extended Kalman Filter

The MEKF is constructed as before, but now there are 9 estimated states instead of 6. The angular velocity estimate is computed from the filter, not directly from the gyro reading. The discrete error-state transition matrix for this case is given by

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{30a}$$

$$\Phi_{11} = I_{3\times3} - [\hat{\boldsymbol{\omega}\times}]\frac{sin(||\hat{\boldsymbol{\omega}}||\Delta t)}{||\hat{\boldsymbol{\omega}}} + [\hat{\boldsymbol{\omega}}\times]^2 \frac{1 - cos(||\hat{\boldsymbol{\omega}}||\Delta t)}{||\hat{\boldsymbol{\omega}}||^2} \tag{30b}$$

$$\Phi_{12} = [\hat{\boldsymbol{\omega}}\times]\frac{1 - cos(||\hat{\boldsymbol{\omega}}||\Delta t)}{||\hat{\boldsymbol{\omega}}||^2} - I_{3\times3}\Delta t - [\hat{\boldsymbol{\omega}}\times]^2 \frac{||\hat{\boldsymbol{\omega}}||\Delta t - sin(||\hat{\boldsymbol{\omega}}||\Delta t)}{||\hat{\boldsymbol{\omega}}||^3} \tag{30c}$$

The measurement vector now includes the gyro measurements as well as the vector attitude measurements:

$$\tilde{\boldsymbol{y}}_k = \begin{bmatrix} \tilde{\boldsymbol{\omega}}_k^T & \tilde{\boldsymbol{b}}_{1,k}^T & \tilde{\boldsymbol{b}}_{2,k}^T & \dots & \tilde{\boldsymbol{b}}_{n,k}^T \end{bmatrix}^T \tag{31}$$

Referring to the gyro measurement model in Eq. (11a), the sensitivity matrix is computed as

$$H_k(\hat{\boldsymbol{x}}_k^-) = \begin{bmatrix} 0_{3\times3} & I_{3\times3} & I_{3\times3} \\ [A(\hat{\boldsymbol{q}}^-)\boldsymbol{r}_1\times] & 0_{3\times3} & 0_{3\times3} \\ [A(\hat{\boldsymbol{q}}^-)\boldsymbol{r}_2\times] & 0_{3\times3} & 0_{3\times3} \\ \vdots & \vdots & \vdots \\ [A(\hat{\boldsymbol{q}}^-)\boldsymbol{r}_n\times] & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \tag{32}$$

The gyro noise $\boldsymbol{\eta}_v$ now appears in the measurement covariance:

$$R_k = \text{diag}\left[\left(\frac{\sigma_v^2}{\Delta t} + \frac{1}{3}\sigma_u^2\Delta t\right) I_{3\times3} \quad \sigma_1^2(I_{3\times3} - (A\boldsymbol{r}_1)(A\boldsymbol{r}_1)^T) \quad \ldots \quad \sigma_n^2(I_{3\times3} - (A\boldsymbol{r}_n)(A\boldsymbol{r}_n)^T\right] \tag{33}$$

There still exists process noise in the bias states due to $\boldsymbol{\eta}_u$. Additionally, there is now process noise in the angular rate states due to accelerations caused by disturbance torques or uncertainty in commanded torques. If this noise is denoted $\boldsymbol{\eta}_\omega$ and is also assumed to be zero-mean Gaussian with spectral density σ_ω, then the discrete-time process noise covariance matrix is

$$Q = \begin{bmatrix} \frac{1}{3}\sigma_\omega^2\Delta t^3 & \frac{1}{2}\sigma_\omega^2\Delta t^2 & 0 \\ \frac{1}{2}\sigma_\omega^2\Delta t^2 & \sigma_\omega^2\Delta t & 0 \\ 0 & 0 & \sigma_u^2\Delta t \end{bmatrix} \tag{34}$$

It is noted that if command torques are applied, these must be integrated forward in the dynamics model, i.e.

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{\tau} \tag{35}$$

where I is the spacecraft's inertia matrix and $\boldsymbol{\tau}$ is the vector of torques. The standard EKF update and covariance propagation can now be applied.

Unscented Filter

As with the dynamic model replacement formulation, an Unscented Filter can also be used. (Reference 14) presents a formulation of the Unscented Filter that estimates attitude, angular rate, and biases, but the attitude is estimated as the vector portion of the error-state quaternion, $\boldsymbol{\delta\varrho}$. As is also the case with the dynamic model replacement filter, this state choice does not work well because large errors or uncertainties can violate the quaternion unit norm constraint. As such, the Unscented Filter with angular rate estimation is reformulated so that the Generalized Rodrigues Parameters of the attitude error state, $\boldsymbol{\delta p}$ are used to estimate the attitude. These parameters follow the same definition as in the dynamic model replacement Unscented Filter. A full derivation is omitted for brevity, but the standard Unscented Filtering process is followed from this point.

CHOICE OF FILTER STATES

Now that the filtering method for both cases has been discussed, it is desired to assess which choice is more appropriate given a choice of sensors and their associated noise parameters. Qualitatively, one expects that dynamic model replacement will perform better when gyro noise is low and disturbance accelerations are high, and angular rate estimation will perform better when gyro noise is high and disturbance accelerations are low.

Ideally, an analytical steady-state covariance for the attitude. Farrenkopf found an analytical steady-state solution for two single-axis cases.[9] The first uses angle and gyro bias as the states ($\boldsymbol{x} = [\theta \;\; \beta]$) and assumes an angle sensor and gyro are used ($\tilde{\boldsymbol{y}} = [\tilde{\theta} \;\; \tilde{\omega}]$). The second assumes that attitude and rate are estimated ($\boldsymbol{x} = \theta \;\; \omega$) but that only an angular sensor is used ($\tilde{y} = \tilde{\omega}$). Markley has extended this for the three-axis case.[15] However, if the state vector is taken to include angle, rate, and bias ($\boldsymbol{x} = [\theta \;\; \omega \;\; \beta]$) and both angular and rate (gyro) measurements are available, no general closed-form solution exists for the steady-state covariance.

Therefore, this paper focuses on presenting results through simulation. Both the EKF and Unscented Filter have been implemented for both the dynamic model replacement and angular rate estimation cases, and performance between the EKF and Unscented Filter has been found to be nearly identical in each case. As such, only EKF results will be presented in this work.

RESULTS

Case 1: Star Tracker With MEMS Gyro, Frequent Star Tracker Updates

The first case analyzed assumes a star tracker and gyro are available, and that the star tracker can provide a reading once per second. The star tracker accuracy is assumed to be $\sigma_{st} = 7.24 \times 10^{-4}$ degrees. The gyro parameters are based on the MAX21000 MEMS gyroscope[16] with $\sigma_v = 0.02$ deg/s and $\sigma_u = 0.0075$ deg/s^2. Disturbance torques are assumed to have a 1σ value of 10^{-5} Nm, which is based of the 6U cubesat GLADOS being developed at the University at Buffalo. Figure 1 shows the performance of the estimator with angular

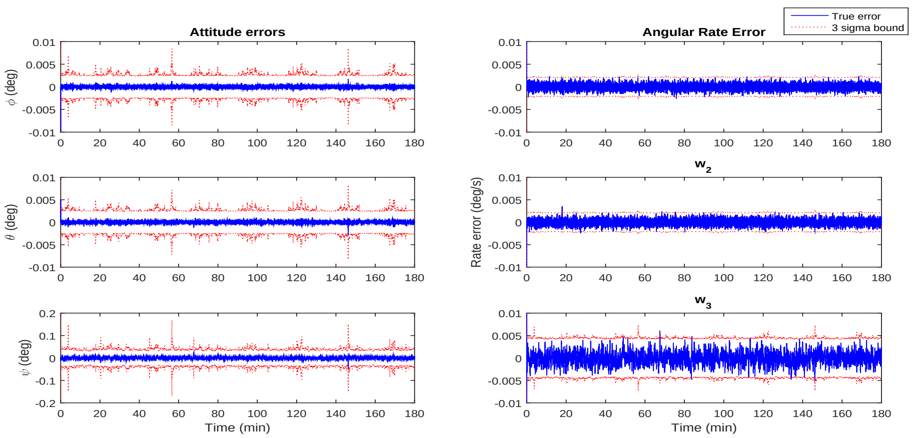

Figure 1. Case 1: Angular Rate Filter

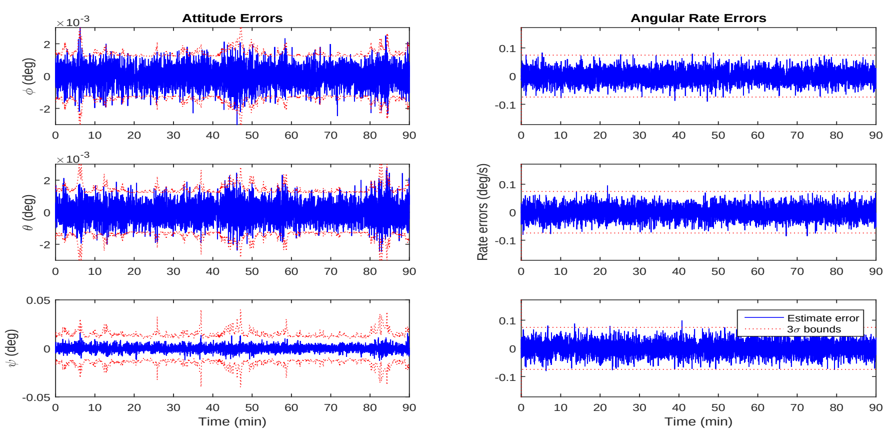

Figure 2. Case 1: Dynamic Model Replacement Filter

rate, while Figure 2 shows the performance of the dynamic model replacement estimator for this case. It is seen that both filters have similar performance in their attitude estimate. The 3σ bounds are smaller in the dynamic model replacement case, with errors of about 0.002 degrees in two axes, compared to about 0.004

degrees for the angular rate estimator. The third axis has a larger uncertainty in both cases, which is explained by the fact that this is the axis which aligns with the star tracker boresight. However, the angular rate estimate is much better in the angular rate estimator – with 3σ bounds of approximately 0.002 degrees in two axes, compared to approximately 0.08 degrees for the dynamic model replacement estimator. This is due to the information contributed by the star tracker to the angular rate estimate. While both filters are found to provide good attitude estimates, if the application requires precise angular rate knowledge, such as tracking a moving object, the angular rate estimator is advantageous.

Case 2: Star Tracker with MEMS Gyro, Infrequent Star Tracker Updates

Now, a similar case is simulated with the limitation that the star tracker update is only available once per minute. This usage case is based off the GLADOS cubesat, which uses a payload camera for star tracking. This camera cannot remain on continuously due to power budget limitations. The noise parameters are assumed the same as in Case 1. Figure 3 shows the performance of the angular rate estimator and Figure

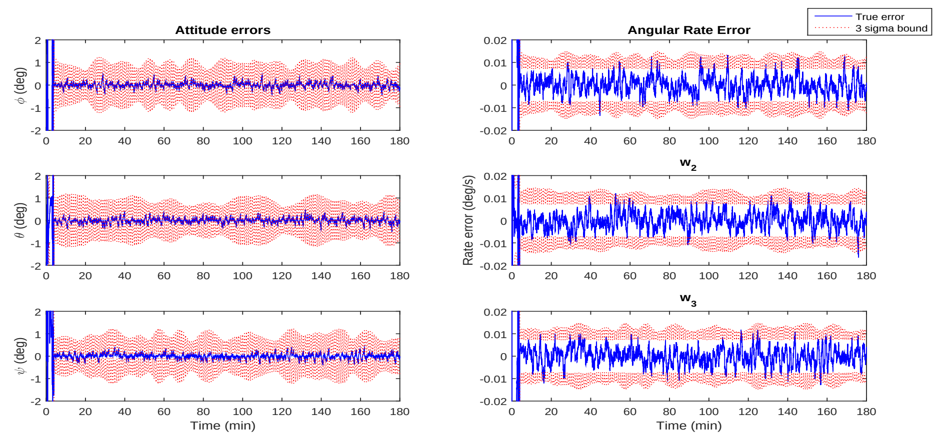

Figure 3. Case 2: Angular Rate Filter

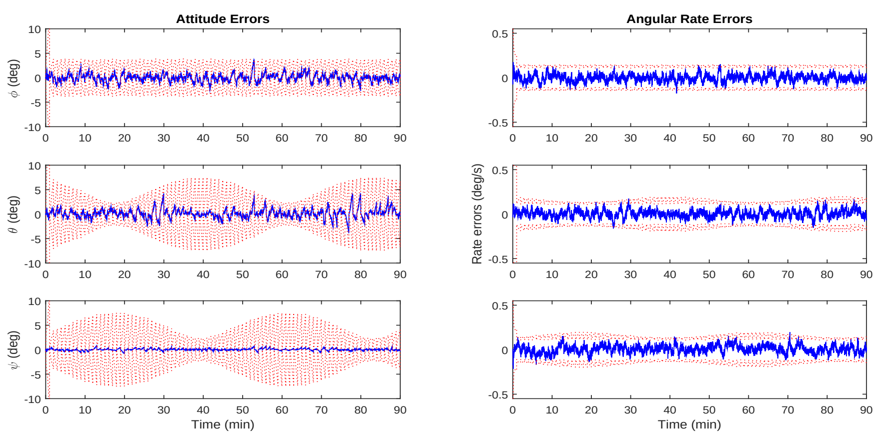

Figure 4. Case 2: Dynamic Model Replacement Filter

4 shows dynamic model replacement. In this case, it is seen that the angular rate estimator performs better for both the attitude and angular rate estimate. Attitude 3σ uncertainty for the angular rate estimator stays between 1 and 2 degrees for each axis for the angular rate estimator, compared to between 5 and 10 degrees

for dynamic model replacement. The angular rate 3σ uncertainties remain less than 0.02 deg/s in each axis for the angular rate estimator, compared to approximately 0.2 deg/s for dynamic model replacement. Note that in both filters, the attitude error and uncertainty increases significantly between star tracker updates and the is reduced when a new star tracker observation is available, so these values would be increase with a lower measurement frequency.

Case 3: Magnetometer, Coarse Sun Sensor, and MEMS Gyro

Now, the case of a magnetometer, coarse sun sensor, and MEMS gyro is simulated. This is a common sensor suite for small satellites without a star tracker. The magnetometer is assumed to have $\sigma_m = 4$ milli-gauss, and the sun sensor is assumed to have $\sigma_{ss} = 0.1$ degrees. An International Geomagnetic Reference Field model is assumed to be available to the spacecraft for determining the inertial direction of the reference vectors. The gyro is assumed to be the same as in Cases 1 and 2. Figure 5 shows the performance of the

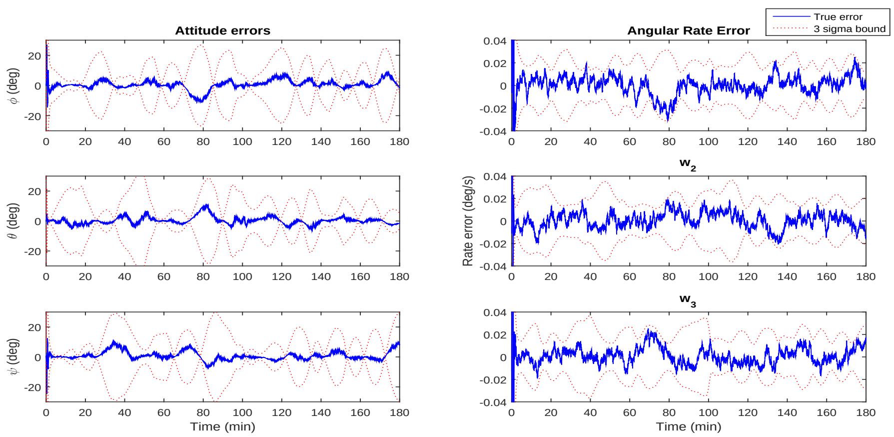

Figure 5. Case 3: Angular Rate Filter

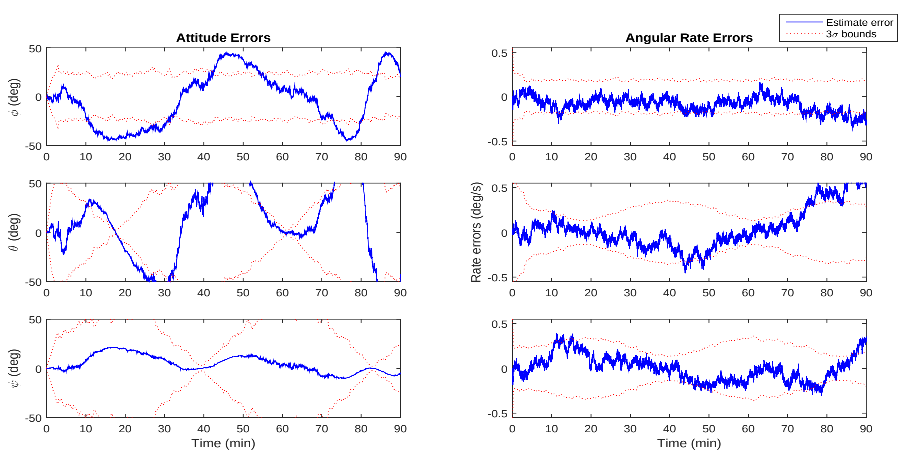

Figure 6. Case 3: Dynamic Model Replacement Filter

angular rate filter and Figure 6 shows the dynamic model replacement filter. The angular rate filter is found to have better performance in both the attitude and angular rate estimates. The 3σ attitude bounds are found to be up to 20 degrees for the angular rate filter, and up to 50 degrees for the dynamic model replacement case. The 3σ angular rate bounds are found to be within 0.04 deg/s for the angular rate filter, compared to up to 0.3 degrees for the dynamic model replacement case.

Case 4: Star Tracker and Mechanical Gyro, Infrequent Star Tracker Updates

Now, use cases are presented assuming that low-noise mechanical gyros are available, as is common on large spacecraft. The gyro noise parameters are assumed to be $\sigma_v = 2\times10^{-5}$ deg/s and $\sigma_u = 2\times10^{-8}$ deg/s^2. This is in a typical range for such gyros.[10] Figure 7 shows the performance for the angular rate filter

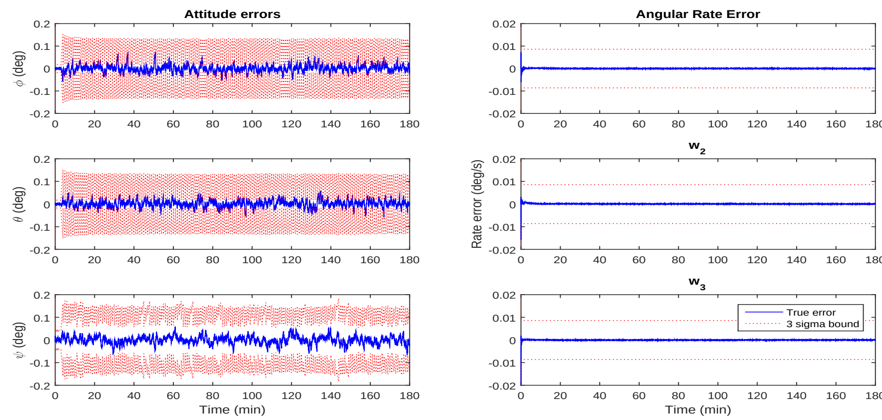

Figure 7. Case 4: Angular Rate Filter

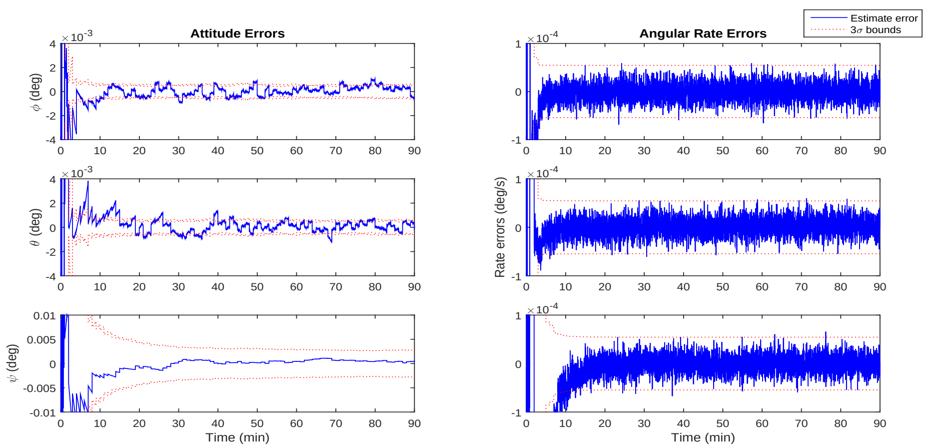

Figure 8. Case 4: Dynamic Model Replacement Filter

and Figure 8 shows the performance for dynamic model replacement. In this case, the dynamic model replacement filter performs better in both the attitude and angular rate estimate, with about two orders of magnitude less uncertainty in both the attitude estimates and angular rate estimates.

Case 5: Magnetometer, Coarse Sun Sensor, Mechanical Gyro

This case revisits Case 3 with the assumption that a mechanical gyro has replaced the MEMS gyro, but no star tracker or other high-precision attitude sensor is available. The magnetometer and sun sensor noise are the same as in Case 3 while the gyro noise is the same as Case 4. Figure 9 shows the performance

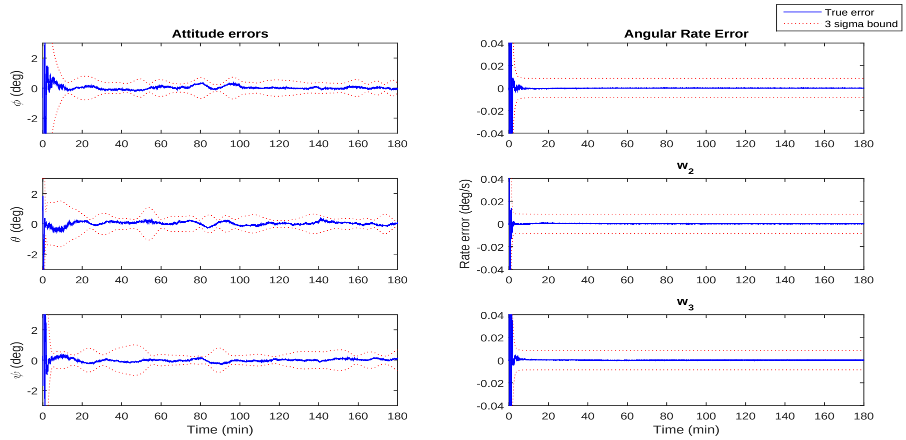

Figure 9. Case 5: Angular rate filter

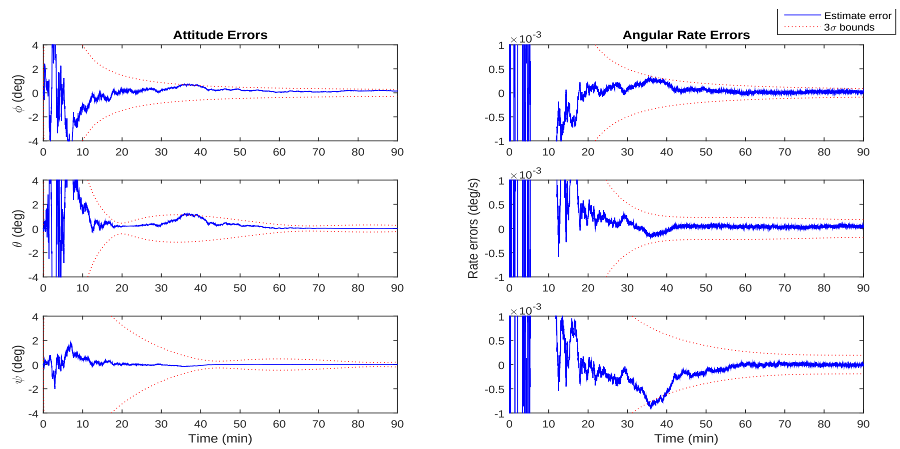

Figure 10. Case 5: Dynamic Model Replacement Filter

of the angular rate estimator and Figure 10 shows the performance of dynamic model replacement for this case. It is seen that the angular rate estimator has faster convergence in the attitude estimate, but that the dynamic model replacement filter eventually converges to a greater accuracy. Additionally, the angular rate estimate has about an order of magnitude less uncertainty in the dynamic model replacement case. The slower convergence of the attitude can be explained by the need for more data to condition an accurate bias estimate in dynamic model replacement, but once the bias estimate uncertainty is low, the attitude error converges further.

CONCULSION

Formulations of the Multiplicative Extended Kalman Filter and Unscented Filter have been presented and implemented for both the case of a filter that estimates attitude, angular rate and gyro bias, and just attitude and gyro bias while using dynamic model replacement for the rate estimates. These filters have been implemented for several use cases. It is seen that given typical noise statistics of MEMS gyros, and expected disturbance torques in the LEO environment, that a filter that estimates the angular rate performs better in every simulated case. This is due to the high noise present in MEMS gyros. It is cautioned that this formulation benefits from the low disturbance torques in space applications, and that application to other systems, such as Unmanned Aerial Vehicles in the atmosphere, may not have the same result. If low-noise mechanical gyros are available, it is seen that dynamic model replacement using the gyro readings performs better in every simulated case. This is due to the ability of the gyro to accurately capture disturbance torques. For small satellite applications using MEMS gyros, it is therefore recommended that the angular rate be estimated in the filter.

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