Estimating Attitude from Signal-to-Noise Ratio at Multiple Ground-Based Receivers

Adonis Pimienta-Peñalver* and John L. Crassidis†
University at Buffalo, State University of New York, Amherst, New York, 14260-4400

This paper presents the implementation of an attitude estimation algorithm to be applied in multiple space- or Earth-based receiver platforms using signal-to-noise ratio from received transmissions from a single broadcasting satellite. A statistical method based on received signal samples is applied to estimate the incoming signal-to-noise ratio, which then facilitates the geometrical derivation of line-of-sight and boresight pointing estimates using geometric relations. The results of this deterministic approach show that at least three independent receivers with non-collinear lines-of-sight are necessary to obtain a three-dimensional boresight solution. Furthermore, an observability study is carried out along with sequential estimation for attitude/maneuver detection. Results indicate a dependence on a-priori knowledge of the bradcaster's RF characteristics, as well the viability of estimating attitude via signal-to-noise ratio as a complement to additional space situational awareness approaches.

I. Introduction

The environmental perils under which satellite systems have to operate are expected to steadily multiply, as globalized world economies move onto even higher levels of interconnectedness, and thus, dependence and demand for orbiting assets. Currently, available tracking systems are thought to catalog only a fraction of the 20,000 bodies larger than 10 cm which orbit the Earth. This is why new methods to observe and characterize resident space objects (RSOs) are necessary to increase the size of the available space catalog, thus enabling operators to plan their missions accordingly. This need for more accurate space situational awareness (SSA) methods is stressed by events such as the 2009 on-orbit collision between the Iridium-33 and Kosmos-2251 communication satellites where, even for arguably large bodies, conventional conjunction analysis procedures failed to predict the accident.

In addition to characterization and classification, knowledge of object-specific parameters of an RSO, such as shape, mass, spin state and solar panel offset, may be used to augment the capabilities of conventional SSA methods, as these are requisite to properly modeling attitude/orbital coupling effects. Novel methods, such as those utilizing light curve data for attribute estimation [1], offer a promising alternative to traditional SSA. However, light curves can be used to sense a change in the angular rate of the target RSO, but not absolute attitude, nor slowly changing attitude maneuvers. Furthermore, by definition, the collection of light curve data is limited to nocturnal operations, making the data sets sparse, and suggesting the need of additional SSA methods to complement the estimation process.

The signal-to-noise ratio (SNR) of an incoming transmission is a variable that, unlike the signal itself depends on the relative position and orientation of the broadcasting and receiving antennae, as well as their radio-frequency (RF) properties. Therefore, the central hypothesis to investigate in this research effort is that the position and attitude information contained in the SNR of an incoming signal from a resident space object (RSO) may be distilled in the form of line-of-sight (LOS) and boresight pointing estimates. The work of [2] and [3] demonstrates the use of SNR from GPS satellite signals to produce LOS vectors toward target from a group of receivers onboard a satellite platform. Reference [4] presents a method to determine SNR level at multiple randomly placed antennas, while [5] describes an SNR estimation method in the frequency domain without a-priori information on incoming signals.

The methods summarized in this paper draw certain elements from the aforementioned papers in order to formulate deterministic and estimation-based algorithms to determine boresight and/or maneuver information from transmitting satellites. A second-and-fourth order estimator, shown in [5], is utilized to obtain estimates of the incoming SNR. This information is then utilized to produce LOS as well as boresight pointing estimates for the broadcasting satellite by way of geometry. Additionally, the viability of utilizing SNR for attitude determination is also investigated in an estimation framework through the application of sequential algorithms and observability studies. Several assumptions are taken throughout this study in order to simplify the problem:

*Postdoctoral Fellow, Department of Mechanical & Aerospace Engineering. Email: arp27@buffalo.edu. Member AIAA
†Samuel P. Capen Chair Professor, Department of Mechanical & Aerospace Engineering. Email: johnc@buffalo.edu. Fellow AIAA.

1) The broadcasting satellite is located in geostationary orbit (GEO): This assumption is taken to reduce complexity for the purposes of the study presented in this paper. Knowing that the broadcasting RSO lies in the geostationary belt significantly reduces the amount of computations required to find an initial guess for LOS from each ground station to the RSO, since the user only needs to perform an azimuthal search across the geostationary belt to produce an initial guess for LOS which may be then refined with a least squares algorithm. Nevertheless, the removal of this assumption will only increase computational load, meaning that the search will have to be made in three dimensions (azimuth , elevation, and altitude). The procedure to obtain an initial guess for LOS is elaborated in section IV.A.
2) The RF model (gain pattern, transmit power, antenna size, etc.) of the satellite is known: The scope of the study presented in this paper is limited to demonstrating a conceptual understanding of the derivation of attitude information from SNR. Although the authors of this paper are of the opinion that the extension of this problem to one where the broadcaster's RF parameters are not known is not an impossible task, it shall be made clear in section IV.B that it is, however, not a trivial one. This paper assumes that these parameters have been previously obtained, which decreases the complexity of the problem significantly.
3) No other interference sources exist aside from atmospheric noise: In order to illustrate the concept of gleaning attitude information from SNR measurements, the RF propagation model utilized in this study is intentionally simple. Further work will have to be done in to introduce realism into the model, which would include accounting for sources of RF interference such as jamming, temperature noise, and weather phenomena, for example.
4) A single continuous link (digital transmission stream) is achieved between the broadcasting satellite and the ground station: Signal loss and multipath effects are not considered in the models used in this paper.

The organization of this paper proceeds as follows. The RF model to simulate SNR values at the receiving ground stations, along with the case study investigated in this paper are presented in section II. Afterwards, section III elaborates on the SNR estimation algorithm, as well as the LOS and boresight estimation procedures. Then, section V delves into the observability of attitude information from SNR measurements, deriving and discussing the expressions pertaining to the available information as a function of attitude. This is followed by brief concluding remarks.

II. Fixed-Beam L-Band Communications Model

This section describes the model that is used in this study to simulate SNR measurements. The analysis starts by defining the power flux density of an isotropic antenna, which radiates uniformly in all directions:

$$\phi_i = \frac{P_t}{4\pi D^2} \tag{1}$$

where P_t denotes the transmitting power and D is the directional antenna diameter. For the case of a directional antenna, which uses a reflector to focus the power into a determined solid angle, the power flux density ϕ would be greater than the isotropic case ϕ_i, since it concentrates all the available power into a small solid angle. The ratio of these two quantities allows to define the antenna gain as

$$G_t = 10\log\left(\frac{\phi}{\phi_i}\right) \tag{2}$$

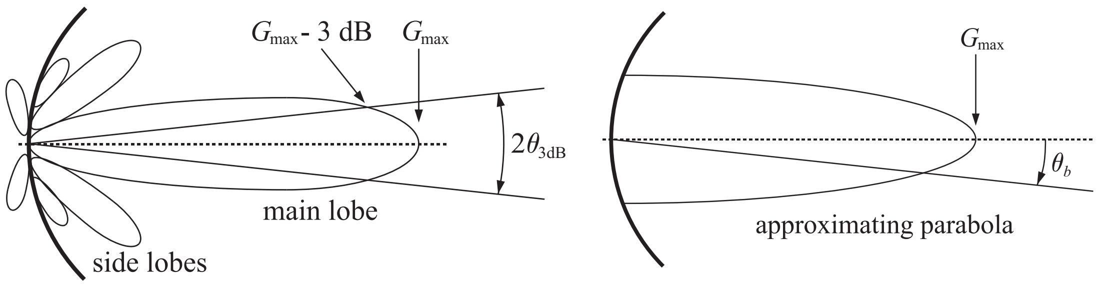
where the index t refers to the transmitting antenna. The equivalent isotropic radiated power (EIRP) is defined as the power which must be radiated by an isotropic radiator in order to achieve the same power flux density as a given directional antenna. It is given in dB as

$$\text{EIRP} = P_t + G_t \tag{3}$$

This concept also extends to receiving antennae. Figure 1a describes the polar representation of a directional antenna gain pattern. There is one main focused lobe, as well as lesser side lobes which project smaller amounts of power. The maximum gain, $G_{\max}$, achievable by any antenna is found on along the boresight, on the main lobe, as shown in Fig. 1a and given in dB by

$$G_{\max} = 10\log\left[\eta\left(\frac{\phi D}{\lambda}\right)^2\right] \tag{4}$$

where η represents the antenna efficiency (usually between 0.55 and 0.65), and the wavelength is given by λ. For a directional parabolic antenna, which is the type used in this study, the term inside the brackets can be approximated by $6.9A/\lambda^2$ [6], where A is the surface area of the parabolic dish.



(a) Directional antenna gain pattern **(b) Approximation of gain pattern**

Fig. 1 RF characteristics of a symmetric directional antenna.

The antenna gain within the main lobe can be approximated by a quadratic decrease with off-boresight angle:

$$G_t = G_{\max} - C\,(\theta_b/\theta_{3\mathrm{dB}})^2 \tag{5}$$

where C is a sizing factor which depends on the antenna gain pattern geometry, θ_b is the off-boresight angle, and $\theta_{3\mathrm{dB}}$ is off-boresight angle at which the gain is 3 dB less than $G_{\max}$. For this study, the gain pattern has been approximated by a parabolic gain pattern as shown in Fig. 1b.

Regarding the receiver antenna, a similar calculation for the direction-dependent gain G_r can be performed. Then, the effects from thermal noise can be accounted for by modeling the temperature noise along the propagation channel. This operation, naturally, would require an accurate model. In this study, we assume low thermal noise $T_r = 38\mathrm{K}$. Then, the figure of merit of the receiver can be calculated by G_r/T_r.

Pending a more rigorous modeling of the propagation channel, including ionspheric effects, multipath distortions, temperature noise, and more, a cursory model of the loss of transmission power can be given by the free space loss over the slant range d, with additional attenuation, ΔL, as a function of the satellite elevation:

$$L_0 = 20\log\left(\frac{4\pi d}{\lambda}\right) \qquad \text{and} \qquad \Delta L = 20\log\left(\frac{d}{h}\right) \tag{6}$$

where h is the altitude of the satellite orbit above the ground station altitude. Thus, a complete measure for the attenuation is additive in nature: $L = L_0 + \Delta L$.

Finally, the SNR can be expressed in dB as follows:

$$\mathrm{SNR} = \mathrm{EIRP} - L + G_r/T_r - k - R_s \tag{7}$$

where k is the Boltzmann constant.

A. Case Study

The ACeS (Asia Cellular Satellite) mobile-to-mobile communications system is utilized in as a case study in the present investigation. The communications bus is carried by the defunct Garuda-1 satellite, which is in GEO. The satellite has a mass of 4500 kg and is sized at 3 × 2.5 × 6 m. An abbreviated list of the system's RF specifications is given in Table 1.

The satellite's orbital state utilized in this study is given by the following TLE:

```
GARUDA 1
1 26089U 00011A 15334.70833333 -.00000218 00000-0 71783+4 0 00007
2 26089 000.0128 195.9086 0000123 075.1421 176.2140 01.00270682000010
```

The simulation time is taken to be the 30th of November of 2015 at 17:00:00 UTC. A group of 8 hypothetical ground stations is selected for the study at locations, shown in Table 2.

III. SNR Estimation

A simple signal model is considered for this study, in which the transmitted signal from the satellite propagates through a White Gaussian Channel. Only a LOS component is assumed to exist between the satellite and the receiving

Table 1 Garuda-1 RF parameters [7]

	Symbol	Value
DC Peak Power	P_t	12 kW
Antenna Diameter	D_t	12 m
Antenna Size Factor	C	10
Data Rate	R_s	2.4 kb/sec
Broadcast Frequency	f	1550 MHz

Table 2 Ground station locations

Location	Latitude (deg)	Longitude (deg)	Altitude (m)
Tainan, Taiwan	23.0149	120.176	19
Kuala Lumpur, Malaysia	3.25006	101.614	53
Surabaya, Indonesia	−7.28252	112.833	5
Darwin, Australia	−12.4501	130.872	23
Coral Bay, Australia	−23.1484	113.773	32
Keeling Islands, Australia	−12.1541	96.8243	1
General Santos, Philippines	6.086357	125.1304	15
Labuan, Malaysia	5.298122	115.207	76

ground station. The effect of multipath and other atmospheric effects are ignored for this simplistic signal model. In the signal model, the k^{th} sample of the signal received at a ground station is given by

$$y_k = x_k + n_k \tag{8}$$

where x_k is the signal transmitted from the satellite and y_k is the signal received at the receiver at sample instance k. The Additive White Gaussian Noise with zero-mean and variance σ^2 is given by n_k. Then, the SNR of the received signal is given by

$$\text{SNR} = \frac{E\{x_k^2\}}{E\{n_k^2\}} \tag{9}$$

where $E\{\cdot\}$ denotes expectation.

Various SNR estimation methods have been proposed in the literature for the Binary Phase Shift Keying signal and other higher modulation schemes. Some of the estimators uses the knowledge of the transmitted signal such as pilots and preambles for the estimation of the SNR and other estimators are based on the samples of the received signal.

In the signal model discussed in this paper, the pilot or preamble symbols are not presumed to be known at the receiver, and hence the SNR estimation is based only on the statistics of the received signal samples y_k. The chosen method to find the SNR is a well-known estimator called Second- and Fourth-Order Moments Estimator (M_2M_4) [5] that relies only on the signal samples received at the ground station. As the name suggests, the algorithm estimates the second-order, $\hat{M}_2$, and fourth-order, $\hat{M}_4$, moments of the received signal's samples as

$$\begin{aligned} \hat{M}_2 &= \frac{1}{L}\sum_{k=1}^{L} |y_k|^2 \\ \hat{M}_4 &= \frac{1}{L}\sum_{k=1}^{L} |y_k|^4 \end{aligned} \tag{10}$$

Then, the signal and noise power estimates, $\hat{S}_{M_2M_4}$, and $\hat{N}_{M_2M_4}$, respectively, which may be used in Eq. (9) to produce

SNR estimates:

$$\hat{S}_{M_2M_4} = \sqrt{2\hat{M}_2^2 - \hat{M}_4} \approx E\{x^2\}$$
$$\hat{N}_{M_2M_4} = \hat{M}_2 - \hat{S}_{M_2M_4} \approx E\{n^2\} \tag{11}$$

Table 3 shows sample results from applying this algorithm, with a mean error of 0.0148 dB.

Table 3 M_2M_4 estimator results

SNR	0	2	4	6	8	10
Estimated SNR	0.02	2.025	4.034	5.98	7.99	10.04

IV. Deterministic Algorithms

Some deterministic algorithms have been developed in order to solve for LOS from each ground station to the broadcaster satellite, and to find the broadcaster's off-boresight angle. Generally, these algorithms are based on geometry, and require knowledge of critical *a-priori* information in order to find solutions. Therefore, deterministic solutions are not always possible.

A. Determination of the Line-of-Sight

If the location of the broadcaster is not previously provided, it may be estimated using LOS from each of the participating ground stations. If the RF parameters of the broadcaster are known, and some a-priori information about attitude is also given, it is possible to perform a search (assuming the target is in GEO) and a least squares refinement to triangulate the position of the target satellite. The algorithm is summarized as follows:

1) Collect estimated SNR at one ground site with known broadcaster's RF parameters and given attitude information (can be boreight direction).
2) Perform an exhaustive search along the visible GEO belt from the receiving ground station, and use the residual error in SNR from the model to converge into a possible location.
3) Perform a nonlinear least squares routine to refine the initial estimate.

Naturally, the accuracy of these steps is dependent on the accuracy of the given information, the resolution of the search step, and the tolerance of the least squares refinement. Furthermore, the position of the target satellite with respect to the ground stations also influences the expected accuracy. Specifically, the LOS estimates degenerate with increasing slant angle from the ground stations. This is shown in Figs. 2 and 3, where it is evident that when the subsatellite point is closer in distance to the ground station locations, the results of the LOS-finding algorithm will improve. This is a result that persists throughout this study. In the future, this will have to be considered when the location of the participating ground stations is chosen.

B. Determination of the Boresight

The process of finding the boresight of the broadcasting antenna uses an inversion of the SNR model in Eq. (7), where the position of the satellite is assumed to be already known or estimated, and the RF parameters of the transmitting antenna are also known a-priori. Once the off-boresight angle, θ_b in Eq. (5), is found by inversion, then the solution is found via geometry. Figure 4 depicts the problem's geometry in 2D, where the upper row of the figure depicts the process of generating the LOS measurements toward the position of the broadcaster satellite based on the off-boresight angles at the known ground station antenna's, ϕ_1 and ϕ_2. This process would be equivalent to a triangulation in three dimensions.

In the 2D representation, after finding the broadcaster's off-boresight angle each ground station, there are two possible boresight directions abscribed to each LOS. The upper right picture in Fig. 4 shows that the off-boresight angles α and β correspond to a pair of isosceles triangles with a common side. Since the boresight of the satellite has to lie along the direction of the edges of the triangles, logic suggests that the solution of this problem is located along the triangle edge that is common to both lines of sight, see the lower right picture in Fig. 4.

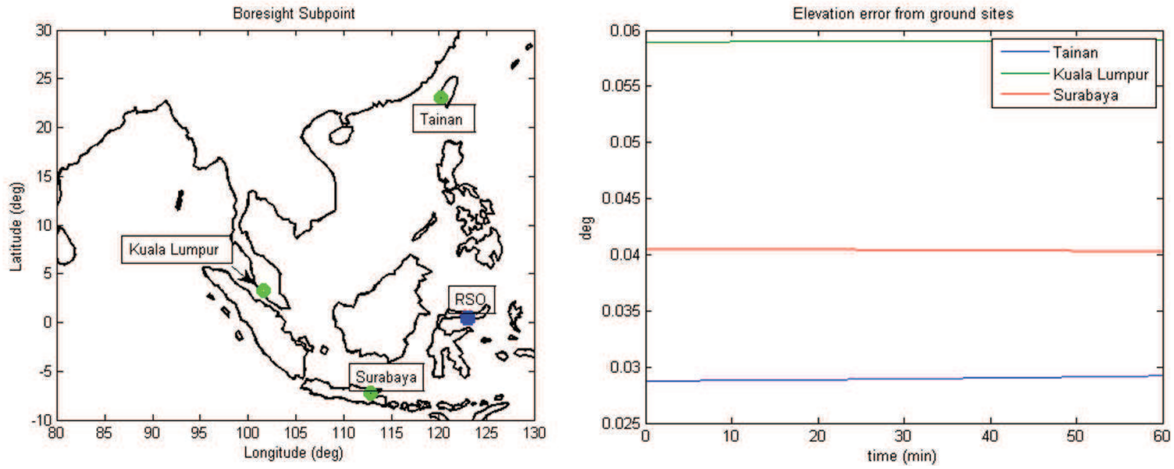


Fig. 2 Configuration # 1. Nadir lat: 0 deg, lon: 123 deg.

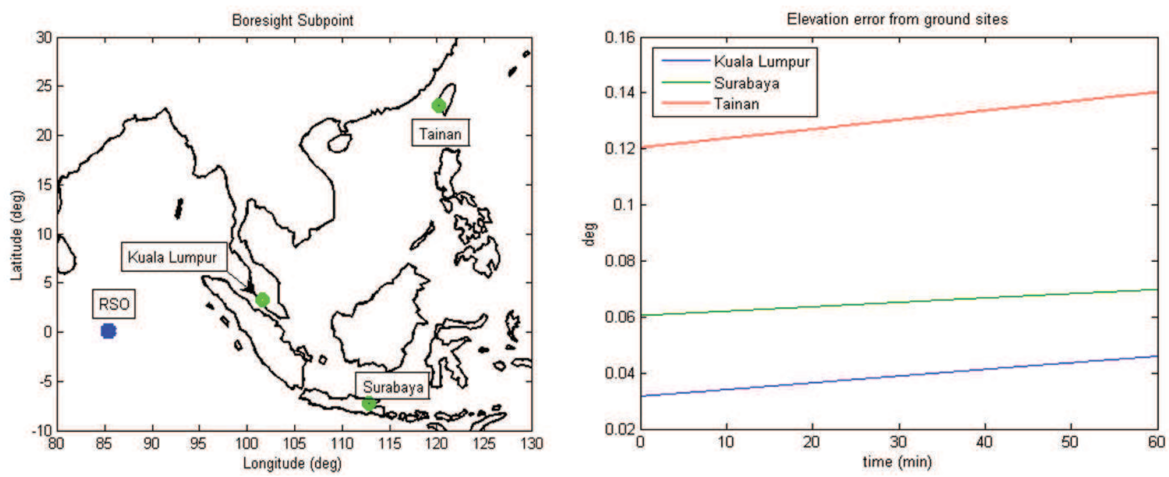


Fig. 3 Configuration # 2. Nadir lat: 0 deg, lon: 85 deg.

Taking this analogy back to 3D suggests that the triangles are susbstituted by cones revolved about the LOS vectors. This is shown in Fig. 5, where the Garuda satellite is triangulated using some nearby ground stations. At least three distinct cones, and therefore, three ground stations, are necessary to find a solution. Mathematically, if ground stations 1, 2, and 3 are considered, the intersections or, in this case, the possible boresight directions, between two cones with the same vertex are given as

$$\eta_1 = \frac{\cos\theta_{b1} - (\mathrm{LOS}_1 \cdot \mathrm{LOS}_2)\cos\theta_{b2}}{1 - (\mathrm{LOS}_1 \cdot \mathrm{LOS}_2)^2} \qquad \eta_2 = \frac{\cos\theta_{b2} - (\mathrm{LOS}_1 \cdot \mathrm{LOS}_2)\cos\theta_{b1}}{1 - (\mathrm{LOS}_1 \cdot \mathrm{LOS}_2)^2}$$
$$\eta_3 = \pm\sqrt{\frac{1 - \eta_1\cos\theta_1 - \eta_2\cos\theta_2}{1 - (\mathrm{LOS}_1 \cdot \mathrm{LOS}_2)^2}} \qquad B_{1,2} = \eta_1\mathrm{LOS}_1 + \eta_2\mathrm{LOS}_2 + \eta_3\,(\mathrm{LOS}_1 \times \mathrm{LOS}_2) \tag{12}$$

where $B_{1,2}$ represents the two possible boresight directions *from the satellite* corresponding to pair (1,2) of ground stations. One only needs to compare this to the solution pairs between (2,3) and (1,3) to find the common solution between all of the pairings, which is shown as the large ‘x’ in Fig. 5.

With perfect knowledge of RSO position (perfect LOS solutions), the boresight determination algorithm error remains at around 10^{-3} degrees. This is only limited by the resolution of the geometric solution. If the wrong broadcasting RF parameters are utilized in the model, then the error of the boresight-finding algorithm can increase significantly, as shown in Fig. 6, where the transmit power and diameter of the broadcasting dish antenna are varied

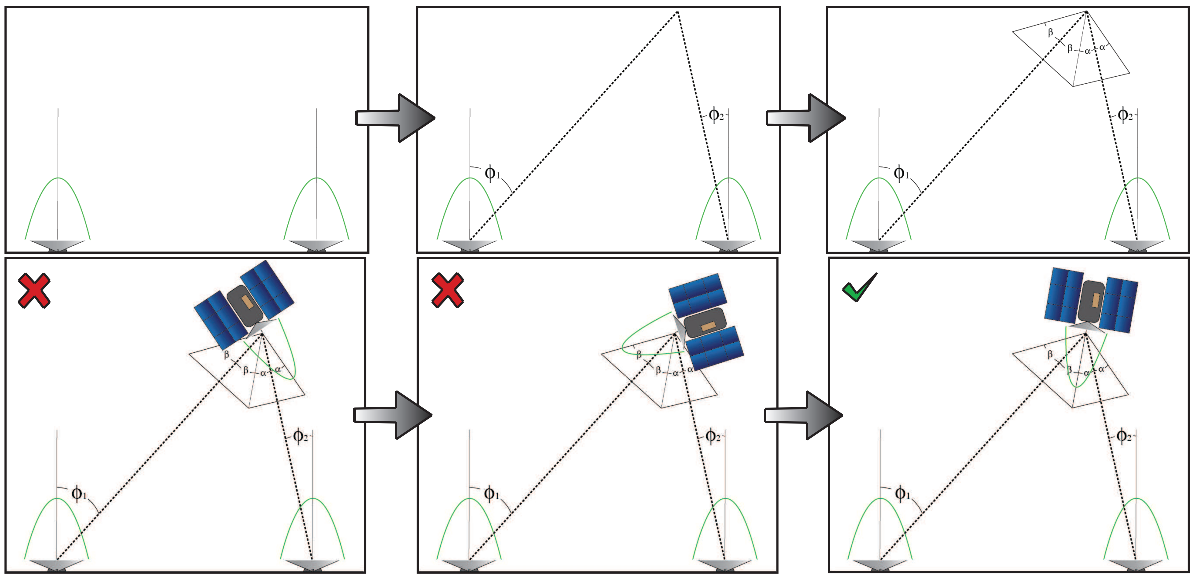


Fig. 4 Finding boresight angle, 2D example.

slightly from 12 kW and 12 m. The appreciably large increase in boresight determination error caused by a discrepancy in the RF parameters means that it is critical that this information is as accurate as possible.

It is to be expected that as the solution is applied over time for a satellite with varying attitude, the boresight determination error will be minimized with increased measurement rate, and number of participating ground stations. The plots in Fig. 7 show the accuracy of the boresight finding algorithm over time by varying the number of receiving ground stations and sampling rate. Note that in the figures, the Garuda-1 satellite attitude follows a westward longitudinal ground track. With only the minimium requirement of ground stations, the solution quickly degenerates as the off-boresight cones become so large that a closed-form geometric solution is no longer guaranteed. Making more ground stations available for the problem, which only needs three ground stations to agree in the boresight solution, means that robustness is introduced to the system. Similarly, if the measurement rate is increased, the difference between consecutive measurements of the incoming signal is minimized. This translates to improvement in the boresight tracking results, since the initial guesses being fed into the nonlinear least squares LOS estimator are closer together and less likely to diverge onto an incorrect solution.

V. Observability

This section presents an observability study by deriving an analysis of the amount and type of attitude information contained within individual SNR estimates. Furthermore, the analysis is *static*, indicating the omission of a study of filter performance within a dynamic situation, and *local*, meaning that the study seeks to determine whether small errors in attitude estimates are observable in the SNR calculations.

Reference [8] cites the use of the Fisher information matrix (FIM) for attitude observability studies. The FIM, which defines the information available in measurement $\tilde{\mathbf{y}}$ from state $\mathbf{x}$, is given by

$$F = E\left\{\left[\frac{\partial}{\partial \mathbf{x}}\ln\left[p\left(\tilde{\mathbf{y}}|\mathbf{x}\right)\right]\right]\left[\frac{\partial}{\partial \mathbf{x}}\ln\left[p\left(\tilde{\mathbf{y}}|\mathbf{x}\right)\right]\right]^{\mathrm{T}}\right\} \tag{13}$$

where $p\left(\tilde{\mathbf{y}}|\mathbf{x}\right)$ is a conditional probability density function of measurements $\tilde{\mathbf{y}}$ given the state $\mathbf{x}$. With a typical measurement process model $\tilde{\mathbf{y}} = \boldsymbol{h}\left(\mathbf{x}\right) + \mathbf{n}$, where the measurement noise $\mathbf{n}$ is a zero-mean Gaussian process with covariance R, the FIM can be expressed as

$$F = \left(\frac{\partial \boldsymbol{h}\left(\mathbf{x}\right)}{\partial \mathbf{x}}\right) R^{-1} \left(\frac{\partial \boldsymbol{h}\left(\mathbf{x}\right)}{\partial \mathbf{x}}\right)^{\mathrm{T}} \tag{14}$$

In this paper, the 3×1 vector of attitude errors, $\boldsymbol{\delta\alpha}$, is considered to be the state vector of interest, $\mathbf{x}$, while the SNR

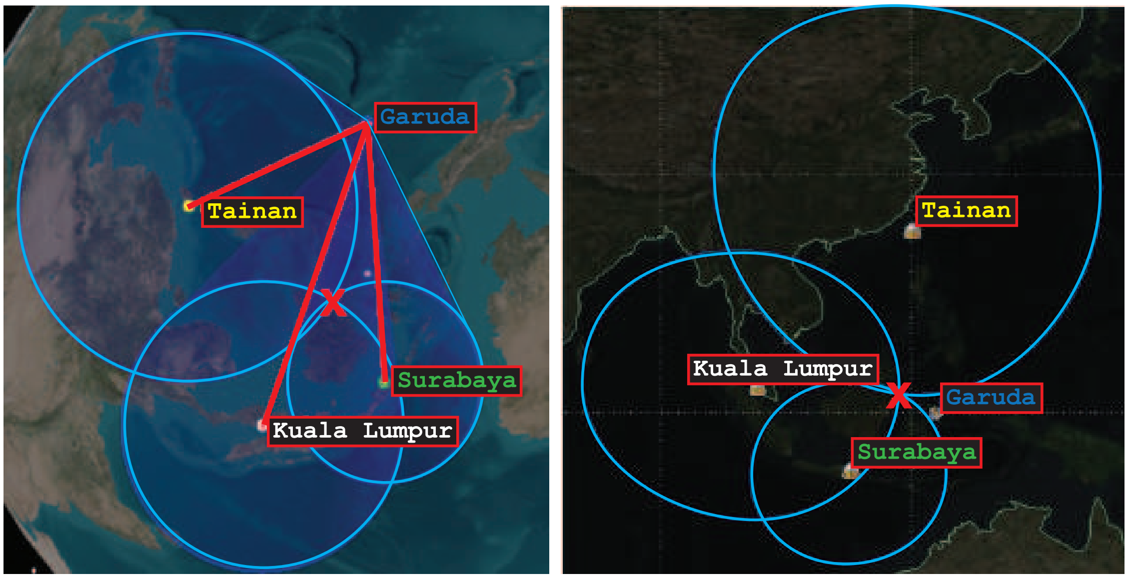


Fig. 5 Boresight cone geometry, 3D example.

results from the M_2M_4 estimator are treated as measurements, $\tilde{\mathbf{y}}$, and are assumed to be distributed in a Gaussian manner with zero mean and σ^2 variance. This means that $\partial\text{SNR}/\partial\boldsymbol{\delta\alpha}$ is a 3×1 vector, and the FIM is of dimension 3×3. As the outer product of two vectors, the FIM will have rank 1 with two zero eigenvalues that correspond to two nullspace eigenvectors, representing the error directions that are unobservable from a single observation of the SNR estimate.

A. Attitude Information

The sensitivity, $\partial\text{SNR}/\partial\boldsymbol{\delta\alpha}$, that plays the role of the partial derivative in Eq. (14), may be derived from the expressions implied in Eq. (7). Assumming that the receiving ground station is pointed at its zenith, the attitude dependence in the SNR model of Section II arises in Eq. (5), which describes the projected gain of the transmitted signal based on off-boresight angle, θ_b. This off-boresight angle can be defined as the angle between the inverted LOS to the ground station and the antenna boresight axis, which in this case, is defined as the nadir direction of the transmitting satellite.

$$\theta_b = \tan^{-1}\left(\frac{\left\|[0\ 0\ 1]_{\mathcal{B}}^{\mathrm{T}} \times (-\text{LOS}_{\mathcal{B}})\right\|}{[0\ 0\ 1]_{\mathcal{B}}^{\mathrm{T}} \cdot (-\text{LOS}_{\mathcal{B}})}\right) \tag{15}$$

The explicit relation to attitude arises when a rotation is needed to transform the LOS vector from the inertial, $\mathcal{I}$, frame to the satellite's body, $\mathcal{B}$, frame. The LOS is found from known coordinates of the ground station, r_{GS}, and the RSO, r_{RSO} as

$$\text{LOS}_{\mathcal{B}} = \hat{A}\left(\frac{r_{\text{RSO}} - r_{\text{GS}}}{\left\|r_{\text{RSO}} - r_{\text{GS}}\right\|}\right) \tag{16}$$

where the estimated attitude, $\hat{A}$, includes the true relative attitude from the inertial frame to the RSO's body frame, A_{true}, and the vector of small attitude errors, $\boldsymbol{\delta\alpha}$.

$$\hat{A} = \exp\left\{-[\boldsymbol{\delta\alpha}\times]\right\} A_{\text{true}} \approx (\mathrm{I} - [\boldsymbol{\delta\alpha}\times])\, A_{\text{true}} \tag{17}$$

Note that $[\boldsymbol{\delta\alpha}\times]$ is the error vector's cross product matrix.

Thus, the chain rule is utilized at Eq. (7):

$$\frac{\partial\text{SNR}}{\partial\boldsymbol{\delta\alpha}} = \frac{\partial\text{SNR}}{\partial\theta_b}\frac{\partial\theta_b}{\partial\boldsymbol{\delta\alpha}} \tag{18}$$

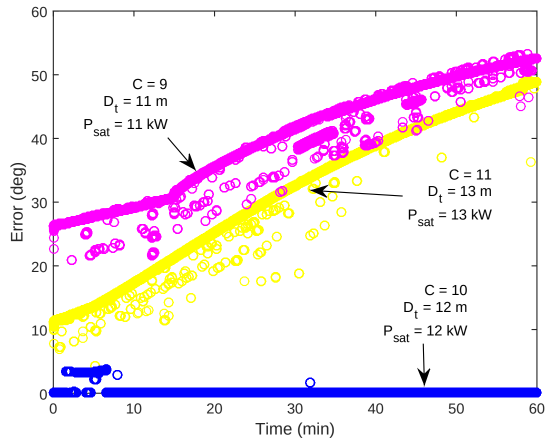


Fig. 6 Boresight determination error with varying RF parameters.

in order to derive the desired sensitivity expression. The first element of the chain rule is given by

$$\frac{\partial \text{SNR}}{\partial \theta_b} = -\frac{2C\theta_b}{\theta_{3dB}^2} \tag{19}$$

while the off-boresight angle sensitivities are given as follows

$$\frac{\partial \theta_b}{\partial \boldsymbol{\delta\alpha_x}} = \frac{\dfrac{(f_1+f_2+f_3)(-f_{16}-f_{17}-f_{18})}{g(-f_7-f_8-f_9)} - \dfrac{g(f_{13}+f_{14}+f_{15})}{(-f_7-f_8-f_9)^2}}{\dfrac{g^2}{(-f_7-f_8-f_9)^2}+1} \tag{20}$$

$$\frac{\partial \theta_b}{\partial \boldsymbol{\delta\alpha_y}} = \frac{\dfrac{(f_4+f_5+f_6)(-f_{16}-f_{17}-f_{18})}{g(-f_7-f_8-f_9)} - \dfrac{g(-f_{10}-f_{11}-f_{12})}{(-f_7-f_8-f_9)^2}}{\dfrac{g^2}{(-f_7-f_8-f_9)^2}+1} \tag{21}$$

$$\frac{\partial \theta_b}{\partial \boldsymbol{\delta\alpha_z}} = \frac{2(f_1+f_2+f_3)(-f_{10}-f_{11}-f_{12}) + 2(f_4+f_5+f_6)(-f_{13}-f_{14}-f_{15})}{2g(-f_7-f_8-f_9)\dfrac{g^2}{(-f_7-f_8-f_9)^2+1}} \tag{22}$$

where the following symbols have been defined for the sake of succinctness:

$$\begin{aligned}
&f_1 = (-a_{11}\delta\alpha_z + a_{31}\delta\alpha_x + a_{21})(-\text{LOS}_{\mathcal{I}x}) && f_2 = (-a_{12}\delta\alpha_z + a_{32}\delta\alpha_x + a_{22})(-\text{LOS}_{\mathcal{I}y}) \\
&f_3 = (-a_{13}\delta\alpha_z + a_{33}\delta\alpha_x + a_{23})(-\text{LOS}_{\mathcal{I}z}) && f_4 = (a_{21}\delta\alpha_z - a_{31}\delta\alpha_y + a_{11})(-\text{LOS}_{\mathcal{I}x}) \\
&f_5 = (a_{22}\delta\alpha_z - a_{32}\delta\alpha_y + a_{12})(-\text{LOS}_{\mathcal{I}y}) && f_6 = (a_{23}\delta\alpha_z - a_{33}\delta\alpha_y + a_{13})(-\text{LOS}_{\mathcal{I}z}) \\
&f_7 = (a_{11}\delta\alpha_y - a_{21}\delta\alpha_x + a_{31})(-\text{LOS}_{\mathcal{I}x}) && f_8 = (a_{12}\delta\alpha_y - a_{22}\delta\alpha_x + a_{32})(-\text{LOS}_{\mathcal{I}y}) \\
&f_9 = (a_{13}\delta\alpha_y - a_{23}\delta\alpha_x + a_{33})(-\text{LOS}_{\mathcal{I}z}) && g = \sqrt{(f_1+f_2+f_3)^2 + (f_4+f_5+f_6)^2}
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
&f_{10} = a_{11}(-\text{LOS}_{\mathcal{I}x}) && f_{11} = a_{12}(-\text{LOS}_{\mathcal{I}y}) && f_{12} = a_{13}(-\text{LOS}_{\mathcal{I}z}) \\
&f_{13} = a_{21}(-\text{LOS}_{\mathcal{I}x}) && f_{14} = a_{22}(-\text{LOS}_{\mathcal{I}y}) && f_{15} = a_{23}(-\text{LOS}_{\mathcal{I}z}) \\
&f_{16} = a_{31}(-\text{LOS}_{\mathcal{I}x}) && f_{17} = a_{32}(-\text{LOS}_{\mathcal{I}y}) && f_{18} = a_{33}(-\text{LOS}_{\mathcal{I}z})
\end{aligned} \tag{24}$$

Also, a_{ij} are the ijth elements of the attitude matrix. It should also be noted that since the scalar SNR "measurements" have zero-mean and σ^2 variance, then the R^{-1} term in Eq. (14) can be substituted by $1/\sigma^2$.

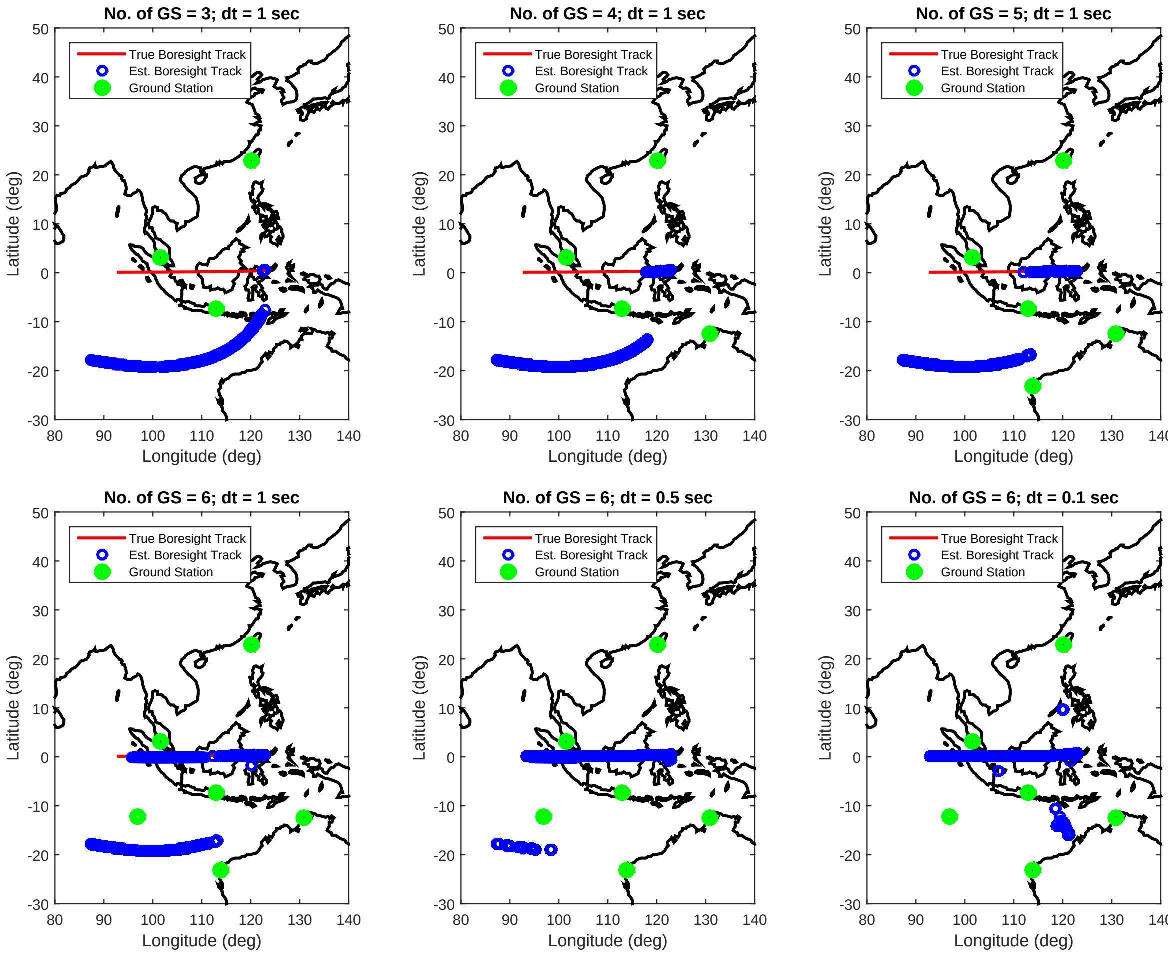


Fig. 7 Boresight error over time with varying ground station number and sampling rate.

As previously mentioned, the FIM is formed by the outer product of a vector with itself, so the nullspace must be made up of two vectors that are orthogonal to $\partial \mathrm{SNR}/\partial \delta \boldsymbol{\alpha}$. It should be noted that the nullspace of the FIM in this study indicates perturbations in attitude that are undetectable from individual measurements of SNR, although, to some degree, these perturbations may be observable using a sequential estimator, as shown in the following subsection. Since the broadcasting RF gain has polar symmetry about the boresight of the antenna, ($[0\ 0\ 1]^{\mathrm{T}}_{\mathcal{B}}$), a rotation of the broadcasting satellite about this direction should bear no weight on the amount of information available, the first nullspace vector should therefore be aligned with the boresight. The second nullspace vector demonstrates a dependence on the LOS to the receiving ground station. This can be seen in Eq. (25), where the x and y components of the LOS vector in the body, $\mathcal{B}$, frame are divided by each other in the first element of the nullspace vector. In essence, the LOS is projected onto the body x-y plane, such that it is orthogonal to the boresight direction. Note that the following expression of the second nullspace vector is obtained by assuming zero attitude error, although accounting for the attitude error does not alter the result:

$$\begin{bmatrix} \dfrac{(r_{\mathrm{RSO}x} - r_{\mathrm{GS}x})\, a_{11} + \left(r_{\mathrm{RSO}y} - r_{\mathrm{GS}y}\right) a_{12} + (r_{\mathrm{RSO}z} - r_{\mathrm{GS}z})\, a_{13}}{(r_{\mathrm{RSO}x} - r_{\mathrm{GS}x})\, a_{21} + \left(r_{\mathrm{RSO}y} - r_{\mathrm{GS}y}\right) a_{22} + (r_{\mathrm{RSO}z} - r_{\mathrm{GS}z})\, a_{23}} \\ 1 \\ 0 \end{bmatrix} \tag{25}$$

Intuition indicates that the amount of attitude information provided by a single estimation of SNR would depend strongly on the attitude of the broadcasting RSO, and therefore on how sensitive SNR is to these changes in orientation relative to the receiving ground station. In this paper, the amount of information available from SNR estimates is parametrized by the only non-trivial singular value of the FIM, which may be calculated by taking the square of the norm of $\partial \text{SNR}/\partial \boldsymbol{\delta\alpha}$ divided by the variance of this estimate, σ^2. Figure 8 shows this quantity as a function of varying attitude around the direction where the boresight is collinear with the LOS for the ground station in Surabaya, Indonesia; in other words, the direction where $\theta_b = 0$. The 'information magnitudes' are plotted in a log scale for easier visualization, with larger values corresponding to more information.

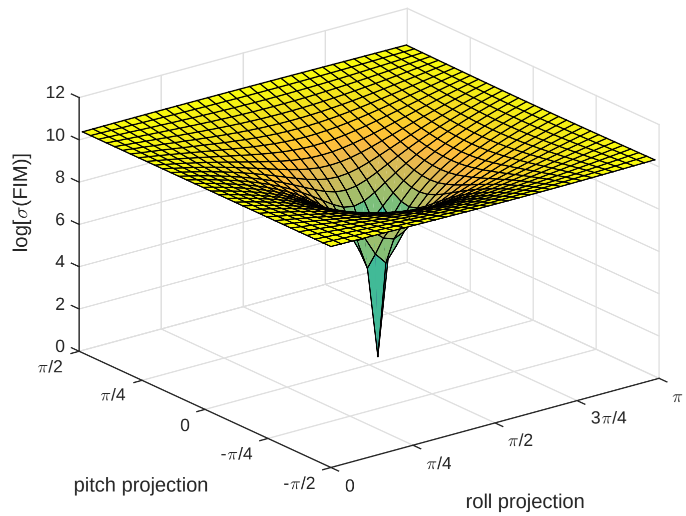


Fig. 8 Information magnitude as a function of attitude.

It is evident from the figure that more information about the attitude of the broadcasting RSO is achievable as the angle between the LOS and the antenna boresight, θ_b increases. This is probably a result of the geometry of the antenna gain pattern: more discernible changes in gain are theoretically possible at more oblique angles from the boresight, since changes in θ_b at angles farther away from the boresight correspond to more pronounced changes in the gain pattern. Note from Eq. (19) that when θ_b is exactly zero, meaning that the broadcaster is pointing directly at the receiving ground station, the sensitivity derivative in Eq. (18) goes to zero. It should be noted, however, that as the off-boresight angle, θ_b, grows, then undesired, and unmodelled, sidelobe effects will distort the broadcaster's gain signature. Therefore, it would be advisable to avoid larger θ_b angles as well as to aim to increase the number of participating ground stations, in order to create some level of redundancy.

B. Sequential Estimation Study

In this section, boresight estimation is demonstrated in a sequential estimation approach using an Unscented Kalman Filter (UKF) architecture [9]. One of the attractive advantages that can be derived from this particular type of sequential filter is that the formulation avoids the derivation of Jacobian matrices [8]. Circumventing the need to calculate the Jacobian of the SNR model might be advantageous in the sense that it helps the user avoid yet another instance of direct mathematical dependence on RF parameters vis-à-vis the demonstrably high sensitivity of the projected boresight solution to *a-priori* knowledge of these values. Nevertheless, the authors of this paper believe that a more conventional Extended Kalman Filter (EKF) solution is entirely achievable.

For the purposes of this paper, the UKF is provided with known RF paramters, as well as position and body rate information, which means that the filter is only tasked with providing estimates for attitude. Furthermore, the formulation follows a multiplicative quaternion representation, as shown in section 6.1.3 of Ref. [8], which defines the true attitude quaternion, $\mathbf{q}^{\text{true}}$, as a product of the estimated quaternion, $\hat{\mathbf{q}}$, and a measure of the body frame attitude error, $\boldsymbol{\delta\vartheta}$, in the following manner:

$$\mathbf{q}^{\text{true}} = \boldsymbol{\delta}\mathbf{q}\,(\boldsymbol{\delta\vartheta}) \otimes \hat{\mathbf{q}} = \begin{bmatrix} q_4 I_3 - [\mathbf{q}_{1:3}\times] & \mathbf{q}_{1:3} \\ -\mathbf{q}_{1:3}^{\mathrm{T}} & q_4 \end{bmatrix} \hat{\mathbf{q}} \tag{26}$$

where $[\mathbf{q}_{1:3}\times]$ stands for the cross-product matrix of the first three components of the quaternion vector, and I_3 stands for the three-element identity matrix.

The orbital parameters at epoch are derived from the provided TLE in section II.A, the satellite's inertia tensor is given by $750I_3$ kg-m^2, while the body rate vector, $\boldsymbol{\omega}$, is defined as a very slow westward slew, as shown in the plots of Fig. 7. The initial attitude quaternion is taken to be

$$\mathbf{q} = [-0.0176892 \quad -0.706875 \quad 0.706868 \quad 0.0187807]^{\mathrm{T}} \tag{27}$$

The initial covariance for attitude is specified at 5 degrees for roll, pitch, yaw. The perceived mean SNR estimation error of 0.0148 dB from the M_2M_4 estimator of section III, is used to define the measurement error matrix, R, which is sized by the number of participating ground stations. The process noise covariance is arbitrarily taken to account for a 0.1 degree variance in the attitude error in all body axis directions. Figure 9 shows filtering results for the aforementioned simulation, where the roll and pitch angles of broadcasting RSO are determined to within a 3σ confidence of ± 10 degrees, while the confidence bounds for the yaw axis is much larger. It is important to note that, since the gain pattern of the broadcasting antenna has a polar symmetry about the boresight, this axial direction is estimated with much less confidence. However, the user should have no interest in correctly determining this yaw rotation, since the boresight pointing projection onto the Earth can be completely defined in terms of roll and pitch.

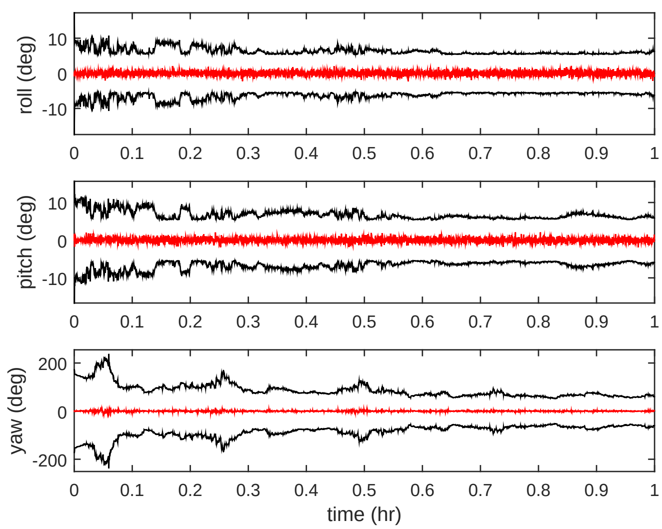


Fig. 9 Unscented filtering errors and 3σ bounds.

Figure 10 shows the same estimation experiment with a 25-degree attitude changing maneuver at 30 minutes. The filter *breaks* shortly after the onset of the maneuver, indicating that a new initial guess for attitude must be taken in order to continue tracking the RSO boresight. This *breaking* is natural, since the assumed body rate suddenly does not correspond to the perceived maneuver. In spite of the failure, this result proves to be an useful tool to sense an attitude changing maneuver in the broadcasting RSO – the user is thereafter tasked with generating a good-enough initial boresight guess to restart the filter.

The simulations of Figures 9 and 10 were taken with three participating ground stations. Subsequent experiments with increasing the number of receivers showed about 0.1 degree of improvement in the 3σ bounds for each additional station. Although this may indicate three participating ground stations to be sufficient, the performance of the estimator as a function of number of receivers is yet to be properly investigated under a more rigorous and realistic filtering simulation.

VI. Conclusions

This paper demonstrates deterministic and estimation-based algorithms to glean attitude information from SNR measurements estimated from incoming signal samples. A deterministic algorithm to find boresight geometrically is demonstrated to be very sensitive to *a-priori* RF parameter information, and to need at least three participating ground stations in order to provide a boresight solution. A static observability analysis is undertaken, deriving the expressions

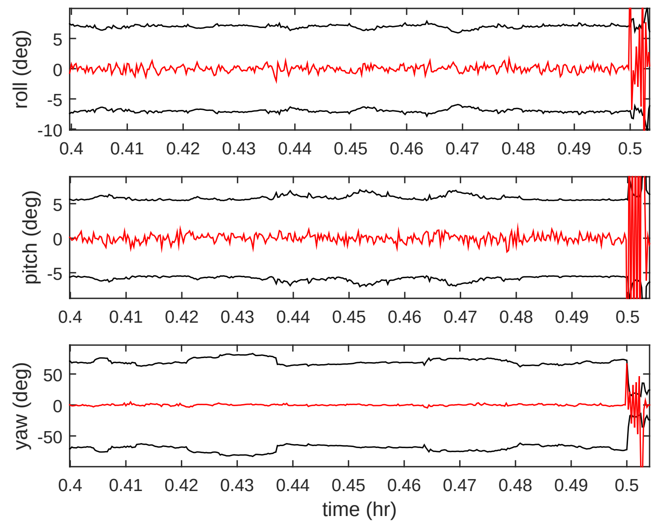


Fig. 10 Unscented filtering errors and 3σ bounds with maneuver.

that constitute the Fisher information matrix and its nullspace. Findings indicate one nullspace vector to lie along the boresight direction, while the second one is given by a projection of the LOS onto the body x-y plane. The amount of attitude information available at any single point in time increases as the angle between the LOS and the antenna boresight increases. A sequential estimation simulation has been shown to demonstrate utilizing SNR measurements to determine RSO attitude. The filter shows lower expected error in the directions that can define the projection of the boresight onto the Earth (roll and pitch), while less confidence is given to the estimated yaw direction. Lastly, the UKF estimation parameters, such as process noise, as well as the SNR model utilized in this paper have been intentionally simplified for the purposes of demonstration. Further work should be directed to enhance the realism of the RF model and the simulation setup.

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